FLUID MECHANICS

LAB MANUAL

1. DESCRIPTION OF THE HYDRAULIC BENCH

The Hydraulic Bench provides facility for performing a number of experiments in hydraulics at laboratory scale. A small SS pump, drawing water from a sump tank, which lies below the bench, delivers to apparatus placed on the bench top. The flow rate is controlled by a valve in the supply line, and is measured by collection of the discharge from the apparatus before return to the sump before recirculation. Around the edge of the bench there is a raised lip, so that any water leaking from the equipment does not spill over the edge, but drains back to the sump through a waste hole provided for the purpose. There is no permanent connection between the bench top and the supporting structure, so the top may be removed easily at any time for inspection of the working parts below.

The measuring tank is supported beneath the bench. A globe valve of normally is provided at the bottom of the tank. The valve can be opened or closed manually.

2. FLOW MEASUREMENT TEST RIG

INTRODUCTION:

The most important class of flow meter is that in which the flow is either accelerated or retarded at the measuring sections by reducing the flow area, and the change in the kinetic energy is measured by recording the pressure difference produced.

APPLICATION FOR FLOW MEASUREMENT:

1) Flow measurement is essential for getting information about the proportional and the amount of materials flowing in or out of a process.
2) The flow rates of utilities like air, water, and steam are required for cost accounting the plant.
3) The flow measurement is highly essential for material balancing of the plant process.
4) The quality of product in continuous processing plant mainly depends upon the correct flow rate of raw materials.

FLOW MEASURING METHODS:

There are two types of flow measurements:

A. Rate of flow: It is the amount of fluid that flows past a given point at any given instant.
B. Total flow: It is the amount of fluid that flows past a given point in a definite time period.

A. RATE OF FLOW MEASURING INSTRUMENTS:

1) Differential pressure meters – (head flow meters)
    a) Orifice meter
    b) Venturi meter
    c) Pitot tube:
    d) Flow nozzle:
2) Variable area flow meters:
    a) Rotameter
    b) Piston type or valve type area meter.
3) Electromagnetic flow meter
4) Flow integrators

TOTAL FLOW MEASURING INSTRUMENT:

1) Positive displacement meters:
   a) Reciprocating piston type flow meters
   b) Rotating disc flow meters
   c) Rotary vane flow meter.
2) Velocity flow meters:
   a) Single jet fan type flow meter
   b) Turbine flow meter
   c) Propeller type flow meter.

B. Flow integrators:

- **RATE OF FLOW MEASURING INSTRUMENTS:**

- Differential pressure type or head flow meters

PRINCIPAL:

Before starting the principal of head flow meters we understand two basic theorems of fluid mechanics. For this consider a flow of incompressible fluid through a pipeline having flow restriction such as orifice as shown in fig.1.

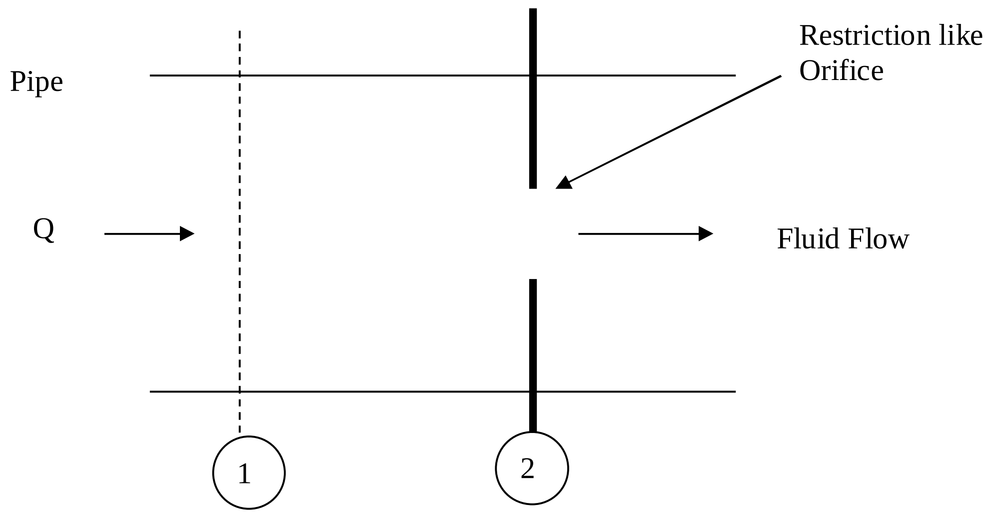


FIG:1 PRINCIPAL OF HEAD FLOW - METERS

CONTINUITY EQUATION:

If ‘Q’ is the volumetric fluid flow rate at the restriction, then continuity equation at section (1) and (2) is stated as

$$\mathbf{Q = A_1V_1 = A_2V_2 = Constant.}$$

Where A and V represents the flow area and flow velocity at the respective sections respectively. Thus according to continuity equations, the product of flow area and flow velocity equals the flow rate of fluid at that section. Since flow area 'A_2' at that restriction is smaller than pipe flow area A_1 at a steady fluid flow the flow velocity V_2 At the restriction must be greater than the velocity 'V_1 ' at the section sufficiently away from the restriction. Thus according to continuity equation, as flow area decreases, flow velocity increases and vice versa.

BERNOULLI'S EQUATION:

This is nothing but energy conservation principal, which states that in steady flow of fluid the sum of potential energy, kinetic energy and pressure energy, remains constant at any section. For above fig. Bernoulli's equation between section (1) and (2) can be written as

Where P1 and P_2 and Z_1 and Z_2 are static pressures and elevations of sections above some datum and is the density of fluid. We have seen that at the restriction section (2) velocity V_2 increases, hence kinetic energy at this section (2) is greater than that at section (1). Accounting to Bernoulli's equation, this increase in velocity energy would be at the cost of decrease in the pressure energy, assuming elevation of the section same. Thus pressure at the restriction is smaller than that at the point sufficiently away from the restriction. Thus at the flow restriction, flow velocity increases with subsequent decrease in static pressure, that developes differential pressure across the restriction. This differential pressure across the restriction varies with flow rate of fluid passing through it. Thus any change in fluid flow rate through the restriction can be measured in terms of differential pressure across it. This is the principal of variable head flow meters.

The most important class of flow meter is that in which the flow is either accelerated or retarded at the measuring sections by reducing the flow area, and the change in the kinetic energy is measured by recording the pressure difference produced.

This class includes:

1. **Pitot tube:**
   In which a small element of fluid is brought to rest at an orifice situated at right angels to the direction of flow. The flow rate is then obtained from the difference between the impact and the static pressure. With this instrument the velocity measured is that of a small filament of fluid.
2. **Orifice meter:**
   In which the fluid is accelerated at a sudden constriction (the orifice) and the pressure developed is then measured. This is a relatively cheap and reliable instrument though the overall pressure drop is high because most of the kinetic energy of the fluid at the orifice is wasted.
3. **Venturi meter:**
   In which the fluid is gradually accelerated to a throat and gradually retarded vas the flow channel is expanded to the pipe size. A high proportion of kinetic energy is thus recovered but the instrument is expensive and bulky.
4. **The Nozzle:**
   In which the fluid is gradually accelerated up to the throat of the instrument but expansion to pipe diameter is sudden as with an orifice. This instrument is again expensive because of the accuracy required over the inlet section.

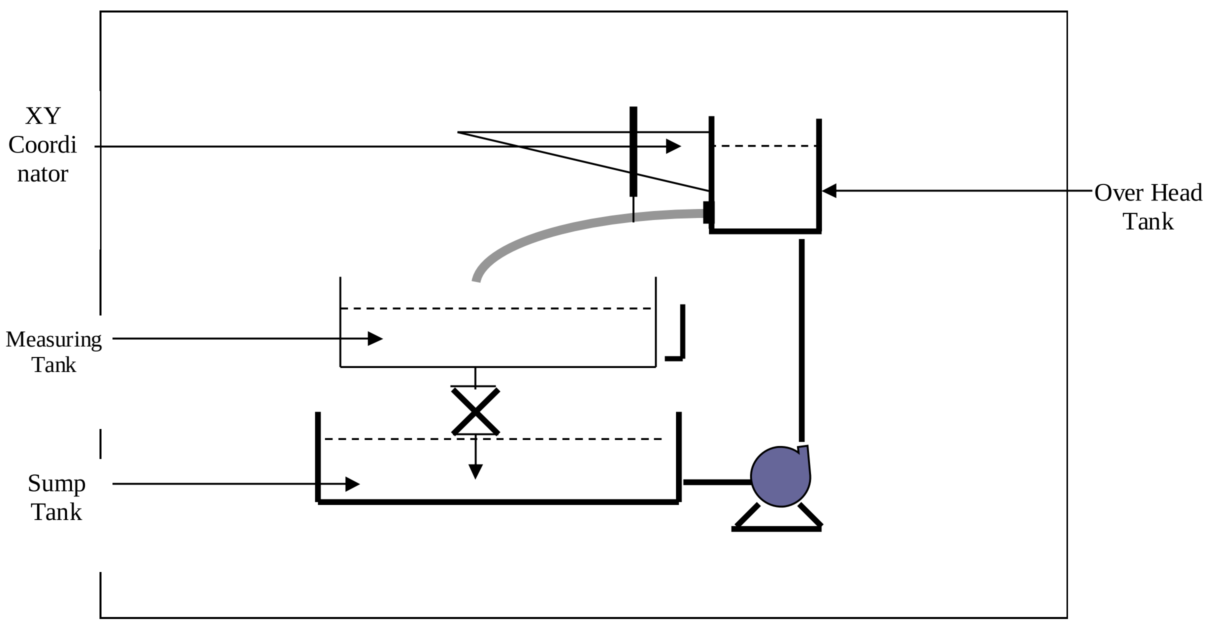
5. **Notch or Weir:**

   In which the fluid flows over the weir so that its kinetic energy is measured by determining the head of the fluid flowing over the weir. This instrument is used in open channel flow and extensively in tray towers, where the height of the weir is adjusted to provide the necessary liquid depth for a given flow.

6. **Variable Area meter or Rotameter:**

   In the meters so far described the area of the constriction or orifice is constant and the drop in pressure is dependent on the rate of flow. In the variable area meter, the drop in pressure is constant and the Flow rate is a function of the constriction. A typical meter of this kind is commonly known as a Rotameter.

<u>Flow Through Orifice:-</u>



DESCRIPTION OF APPARATUS:

The water is fed from the bottom Sump tank. The orifice under test fitted into the base of the tank.

THEORY OF FLOW THROUGH AN ORIFICE :

Let the elevation of the water surface above the plane of the contracted section be H_0. A typical streamline of the flow runs from some point S in the surface to the point T of the contracted section. Then, according to Bernoulli's theorem, in the absence of loss along the streamline, the total had is constant from S to T, so

$$\frac{u_s^2}{2g} + \frac{p_s}{w} + z_s = \frac{u_t^2}{2g} + \frac{p_t}{w} + z_t$$

If the surface area of the tank is large in comparison with that of the orifice, then u_s will be negligible. Also since the pressure is atmospheric at both S and T, $p_s = p_t$. So, noting that:

$$z_t - z_s = H_0$$

by substituting in Bernoulli's equation we find the velocity u_t at T to be:

$$u_t = \sqrt{2gH_0}$$

This is the ideal velocity at T, on the basis of constant total head along the streamline. The same result applies to all streamlines of the flow, so choosing the symbol u_0 to denote this ideal velocity, we find

$$u_0 = \sqrt{2gH_0} \qquad \textbf{(1)}$$

The ideal velocity u_0 at the contracted section is seen to be that which is acquired by a body falling from rest under gravity through a height H_0. This result is often referred to as Torricelli's Theorem.

Because there is some loss of total head, the actual velocity u_c at the contracted section will be rather smaller than the ideal.

The discharge coefficient C_d is defined as the ratio of the actual discharge to the ideal discharge Q_0, which would take place of it if the jet were to discharge at the ideal velocity without reduction of area. The actual discharge Q is

$$Q = A_c u_c \quad \textbf{(2)}$$

Now, if the jet were to discharge at the ideal velocity u_0 over the orifice area A_0 the ideal discharge Q_0 would be

$$Q_0 = u_0 A_0 = \sqrt{2gH_0}\,A_0 \quad \textbf{(3)}$$

So the discharge coefficient may be written in the form

$$C_d = \frac{Q}{Q_0} = \frac{Q}{A_0\sqrt{2gH_0}} \quad \textbf{(4)}$$

We may think of this equation as showing the discharge being reduced from its ideal value in the ratio C_d by two influences. First, fluid viscosity reduced the velocity from its ideal value in the ratio C_u. Secondly the effective cross sectional area of flow at the contracted section is less than at the orifice in the ratio C_c. It is the product of these two effects which leads to flow reduction in the ratio C_d. i.e. $C_d = C_c * C_u$.

PROCEDURE :

⇒ Fill the storage tank with the water.

⇒ Connect the plug to the power supply cable.

⇒ Make sure that bypass valve is fully open and the control valve is fully closed.

⇒ Switch On the pump.

⇒ Keep the delivery valve open and close the bypass valve to have maximum flow rate through the open Tank.

⇒ Open the delivery valve and allow the water to spill over the Overflow line.

⇒ Keep the drain valve of the collection tank open till its time to start collecting the water.

⇒ Close the drain valve of the collection tank and note down the initial level of the water in the collection tank and also the Pressure drop.

⇒ Collect a known quantity of water in the collection tank and note down the time required for the same.

- ⇒ Change the flow rate of water through the open Tank with the help of delivery valve and repeat the above procedure.
- ⇒ Take about 5-6 readings for different flow rates.

OBSERVATION:

CONCLUSION:

RESULTS AND CALCULATIONS:

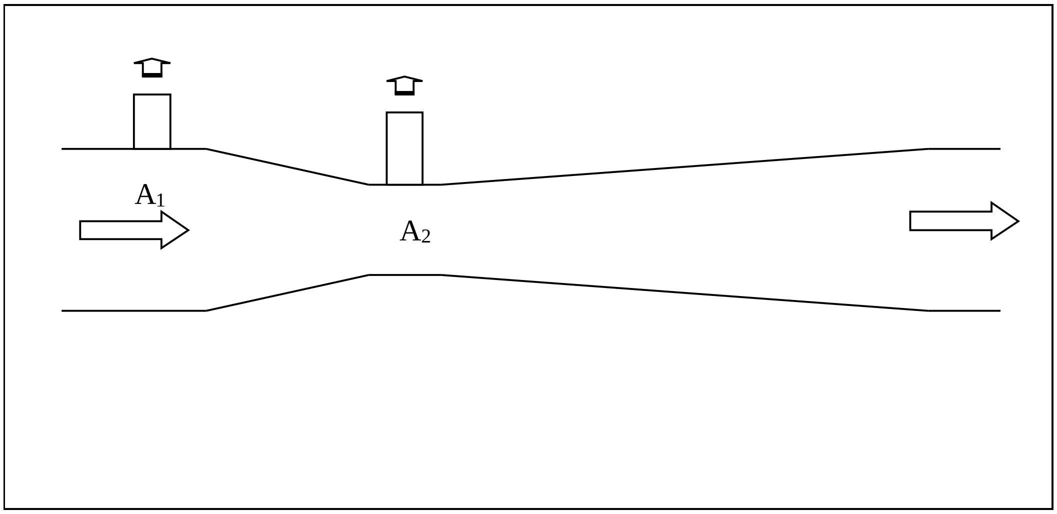
- ⇒ Type of Orifice : Sharp Edged
- ⇒ Diameter of orifice :D_0 = cm
- ⇒ Cross sectional area of orifice :A_0 = cm^2
- ⇒ Time required to collect water := s
- ⇒ Discharge rate Q := cm^3/ s
- ⇒ Coefficient of Discharge $C_d = \dfrac{Q}{A_0\sqrt{2gH_0}}$

V cm^3	**t (s)**	**Initial H in Manometer (cm)**	**Final H in Manometer (cm)**	**Manometer Diff. in (cm)**

FLOW THROUGH VENTURI METER:

Introduction:

The Venturi tube (after Giovanni Battista Venturi (1746-1822), who performed experiments on flow in tapered tubes, including the convergent-divergent combination used in the Venturi meter of today) is a device which has been used over many years for measuring the rate of flow along a pipe. As may be seen from figure, it consists essentially of a tapering contraction section, along which the fluid accelerates towards a short cylindrical throat, followed by a section which diverges gently back to the original diameter. (Such a slowly diverging section is frequently referred to as a diffuser). As the velocity increases from the inlet section to the throat, there is a fall in pressure, the magnitude of which depends on the rate of flow. The flow rate may therefore be inferred from the difference in pressure, as measured by manometers placed between the upstream end and the throat. Such a unit is referred to as a Venturi flow meter.



Another way of metering the flow would be to insert a sharp edged orifice into the pipe; the differential pressure produced by flow through the orifice may similarly be used to infer the flow rate. Such a orifice meter has the advantage of simplicity and cheapness. In comparison with the Venturi tube, however, it causes a greater loss of total head than does a corresponding Venturi meter. This is because much of the velocity head at the throat is recovered as the fluid decelerates in the diffuser. Indeed, the differential manometric head from inlet to the throat can be several times as great as the loss of total head across the whole device.

Description of Apparatus:

Venturi meter is manufactured from Brass material. Water is admitted from the bench supply valve and passes through a flexible hose & GI pipe into the meter. Beyond the control valve, which is mounted just downstream of the meter, a further flexible hose leads to the measuring tank. The manometer tapings in the wall of the Venturi tube are connected to vertical manometer tubes, mounted in front of a scale marked in millimeters. The whole assembly is supported on a base.

Theory of the Venturi Meter:

Consider flow of an incompressible, inviscid fluid through the convergent – divergent Venturi tube shown in Figure. The cross sectional area at the upstream section 1 is A_1, at the throat section 2 is A_2, and at any other arbitrary section n is A_n. Manometer tapings at these sections register h_1 and h_2. Assume that both the velocity and the pressure head are constant over each of the sections considered. This amounts to assuming the flow to be one dimensional, so that the velocity and the pressure head vary only in the direction of the tube length. We may then treat the convergent – divergent pipe as a stream tube, along which Bernoulli's theorem states.

$$\frac{u_1^2}{2g} + h_1 = \frac{u_2^2}{2g} + h_2 \qquad \textbf{(2.1)}$$

in which u_1 and u_2 are the flow velocities at sections 1and 2.

The equation of continuity is

$$u_1 A_1 = u_2 A_2 = Q \qquad \textbf{(2.2)}$$

in which Q denotes the rate of volume flow or discharge. Substituting in Equation (2.1) for u_1 from Equation (2.2), gives

$$\frac{u_2^2}{2g}\left[\frac{A_2}{A_1}\right]^2 + h_1 = \frac{u_2^2}{2g} + h_2$$

and solving this for the velocity u_2 in the throat leads to

$$u_2 = \sqrt{\frac{2g(h_1 - h_2)}{1 - (A_2 / A_1)^2}}$$

The rate of flow Q is found by multiplying the throat velocity u_2 by the cross sectional area A_2 at the throat, giving

$$Q = A_2 \sqrt{\frac{2g(h_1 - h_2)}{1 - (A_2 / A_1)^2}} \quad \textbf{(2.3)}$$

This is the **ideal** discharge rate, obtained by assuming inviscid, one-dimensional flow. In practice, there is some loss of head between sections 1 and 2. Also, the velocity is not absolutely constant across either of these sections. As a result, the actual values of Q fall a little short of those given by Equation (1.3). It is customary to allow for this by writing,

$$Q = CA_2 \sqrt{\frac{2g(h_1 - h_2)}{1 - (A_2 / A_1)^2}} \quad \textbf{(2.4)}$$

in which C is known as the discharge coefficient or simply the coefficient of the Venturi meter. Its value, which usually lies between 0.92 and 0.99, is established by experiment. It varies from one meter to another, and even for a given meter it may vary slightly with the flow rate.

Experimental Procedure:

The apparatus is first leveled. This is done by opening both the bench supply valve and the control valve downstream of the meter, so as to allow water to flow for a few seconds to clear air pockets from the supply hose. The control valve is then gradually closed, while keeping the air vent cocks on the manometer open, so subjecting the venturi tube to a gradually increasing pressure, which causes water to rise up the tubes of the manometer, thereby removing the air contained in the manometer. When the water start flowing our of the air vent cocks close both the cocks simultaneously. Make sure that height of mercury is same in both the limbs of the manometer.

To establish the meter coefficient, measurements are made of a set of differential heads ($h_2 - h_1$) and flow rates Q. The first reading should be taken with the maximum possible value of (h_2-h_1). This condition is obtained by gradually opening both the bench valve and control valve in turn.

Successive opening of either valve will increase both the flow and the difference between h_1 and h_2. Opening of the bench valve is accompanied by a general rise in levels in the manometer, while opening the control valve has the opposite effect. By judiciously balancing the setting of the two valves, the required condition may be obtained.

The rate of flow is found by timing the collection of a known amount of water in the weighing tank, in the meantime values of h_2-h_1 being read from the manometer scale. Similar readings are taken over a series of reducing values of (h_2-h_1), roughly equally spaced over the available range from 60 mm to zero. About 5 readings should suffice.

Observation:

1. Diameter of pipe D1 =
2. Diameter of throat D2 =
3. Area of cross section of pipe $A_1 = \pi/4 * D_1^2$

4. Area of cross section of throat $A_2 = \pi/4 * D_2^2$
5. Area ratio A_2/A_1 =
6. Area of measuring tank A =
7. Length of diffuser =
8. Length of overall Venturimeter =

OBSERVATION TABLE:

Sr.No.	1	2	3	4
Manometer Reading, cm				
Diff. Pressure Δh m Hg				
Diff. Pressure Δh mWc				
Initial Reading of tank H_1 cm				
Final Reading of tank H_2 cm				
Time of collection t sec.				
Volumetric Flowrate Q=V/t, m^3/s				
Co-efficient of Discharge C_d				
Log Q				
Log h				

CALCULATION FOR READING 1:

1. Difference in Manometer Limbs =

   (h1 – h2)

2, Volumetric Flowrate Q

Q = (H2 – H1) * Area of measuring tank / time of collection.

3. Coefficient of discharge Cd,

$$Q = CA_2 \sqrt{\frac{2g(h_1 - h_2)}{1 - (A_2 / A_1)^2}}$$

In above equation $A_2 \left(\frac{2g}{1 - (A_2 / A_1)^2} \right)^{1/2}$ is constant

Its value $A_2 \left(\frac{2g}{1 - (A_2 / A_1)^2} \right)^{1/2}$

$\therefore$ Cd = Q / [9.7585 * 10^{-4} * (Δh) $^{1/2]}$

SAMPLE CALCULATION FOR VENTURIMETER

Observations:

1) Diameter of pipe D1 = 0.025 m
2) Diameter of throat D2 = 0.016 m
3) Area of cross section of pipe $A_1 = \pi/4 * D_1^2$

   1. $= 4.91 * 10^{-4}$ sq.m.

4) Area of cross section of throat $A_2 = \pi/4 * D_2^2$

   1. $= 2.01 * 10^{-4}$

5) Area ratio A_2/A_1 = 0.4094
6) Area of measuring tank A = 0.16 sq. m.
7) Length of diffuser = 0.089 m
8) Length of overall Venturimeter = 0.130 m

OBSERVATION TABLE:

Sr.No.	1	2	3	4
Manometer Reading, cm				
Diff. Pressure Δh m Hg				
Diff. Pressure Δh mWc				
Initial Reading of tank H_1 cm				
Final Reading of tank H_2 cm				
Time of collection t sec.				
Volumetric Flowrate Q=V/t, m^3/s				
Co-efficient of Discharge C_d				
Log Q				
Log h				

OBSERVATION:

Venturimeter:

Diameter at inlet	D_1	= 28 mm
Diameter at throat	D_2	= 12 mm
Cross sectional area at throat	$A_2 = \pi \frac{D_2^2}{4}$	= $1.113*10^{-4}$ m^2
Area ratio, throat to inlet	$\frac{A_2}{A_1} = \left(\frac{D_2}{D_1}\right)^2 =$	= 0.18

Sr. No.		**1**	**2**
Manometer Reading m	**X1**		
	X2		
	ΔX		
***Diff. Pressure Δh* mwc**			
Velocity Va m/s			
Volumetric Flowrate Q m^3/s			
Co-efficient of Discharge C_D			
Log h			
Log Q			

FLOW THROUGH ROTAMETER

Introduction:

In Pitot tube, Orifice, Venturi, Nozzle and Notches or Weir type flow meter the fluid is either accelerated or retarded at the measuring section by reducing the flow area, and the change in the kinetic energy is measured by recording the pressure difference produced. In the variable area meter, the drop in pressure is constant and the flow rate is a function of the area of the constriction.

A typical meter of this kind, which is commonly as a Rotameter consists of a tapered tube with the smallest diameter at the bottom. The tube contains a freely moving float, which rests on a stop at the base of the tube. When the fluid is flowing, the float rises until its weight is balanced by the up thrust of the fluid, its position then indicating the rate of flow. The pressure difference across the float is equal to its weight divided by its maximum cross sectional area in a horizontal plane. The area of annulus is the annulus formed between the float and the wall of the tube.

Experimental Setup:

The setup consist of 3-30 LPM glass tube Rotameter, 0.50 HP Monoblock pump with supply and bypass line, .

The water is pumped from the sump tank through a horizontal pipe. The rate of flow of water is controlled by supply valve and bypass valve. Rotameter is connected to the sections of pipe by threads.

Procedure:

- Fill the sump tank with water.
- Switch on the pump and regulate the flow with the help of supply and bypass valve.
- Allow the unit to come to a steady state as indicated by constant position of the float in Rotameter.
- After steady state is reached record the flow rate indicated on the scale of the Rotameter.

- Take the reading for 1 min and find the length of water in side glass for different flow rates.
- Repeat the above for at least six- seven runs starting from the minimum flow rate and going to a maximum value.

Calculation of Flow rate:

Example:

Flow rate = l * b* h /time

= (3.1*39.5 * 39.5 * 1000) / (100 * 100 * 100 *1)

= 4.83 LPM

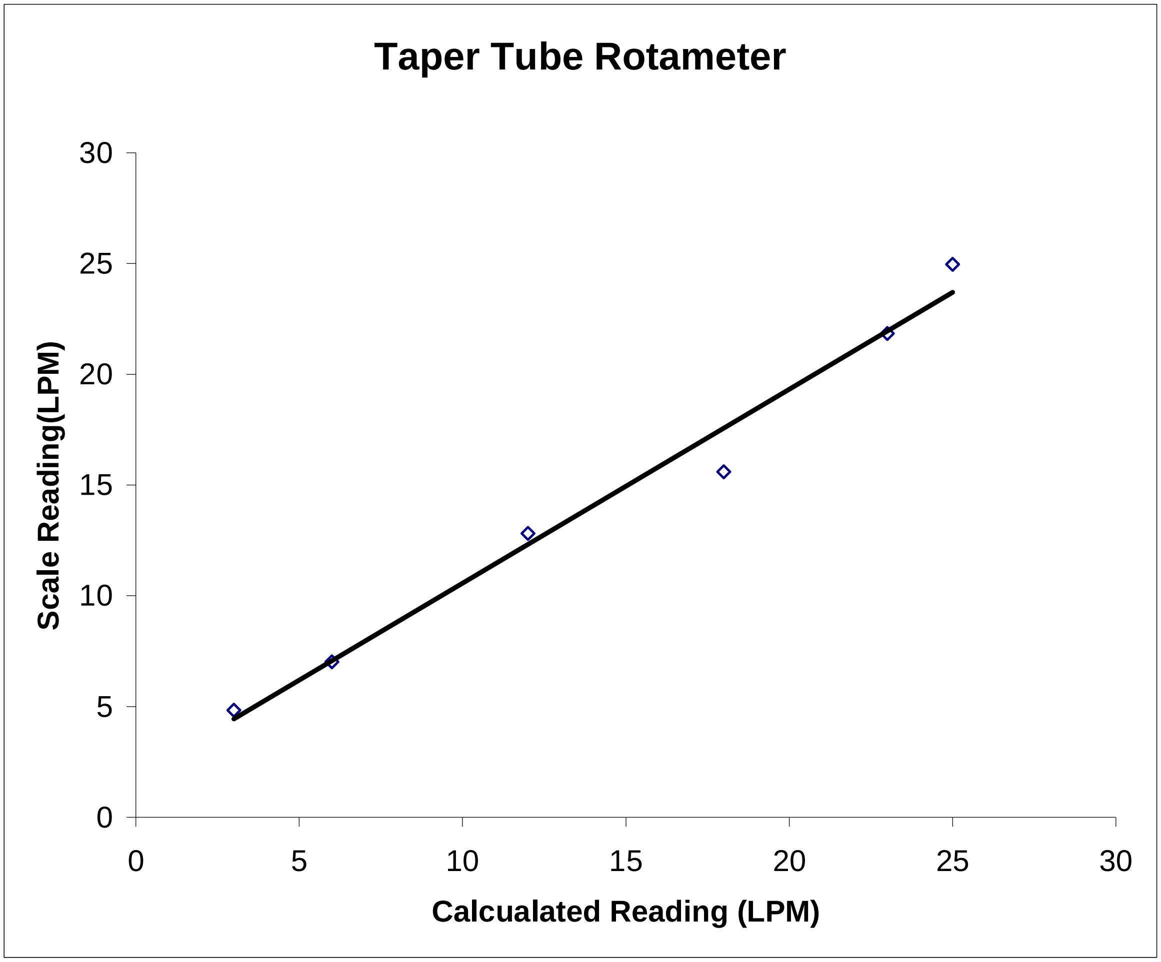
Similarly for others

Observation & Calculation:

Sr. No.	Rotameter Scale reading (LPM)	Measuring Length in 1 minute (cm)	Calculated Flow Rate (LPM)
1	3	3.1	4.83
2	6	4.5	7.02
3	12	8.0	12.82
4	18	10.0	15.60
5	23	14.0	21.84
6	25	16.0	24.96

Graph:

Plot Scale Reading (LPM) Vs. Calculated Value (LPM).


Taper Tube Rotameter
Scale Reading(LPM)
30
25
20
15
10
5
0
0
5
10
15
20
25
30
Calcualated Reading (LPM)

3. Flow over Weirs & notches

Introduction:

Weirs are commonly used to regulate flow in rivers and in other open channels. The purpose is often to maintain water depth for some purpose such as navigation, but a weir may also be used to measure the flow rate.

In many cases, the rate of flow over the weir depends solely on the water level just upstream of the weir (the relationship between flow rate and water level being sometimes known as the “rating curve”). However, the water level downstream of the weir may rise sufficiently as to affect the conditions of flow, so that the flow rate now becomes a function not only of the upstream water level but also of the water level downstream. The weir is then referred to as being “suppressed” or “drowned”.

The cross-section of a weir is usually determined by considerations of strength and stability in relation to the conditions of the site, and availability of materials. The crest is frequently rounded or broad. For such weirs, the flow usually remains attached to the downstream surface. In the case of the sharp crested weir, however, the flow separates at the crest to form a curved jet which plunges into the downstream pool. In plan view, the weir may be straight or curved to suit site conditions, or on grounds of aesthetics. Frequently, the crest level is not uniform along the whole of the length. For instance, just part of the whole length may used to carry normal flow, the remainder of the weir having a higher crest, so that it comes into use only at higher flow rates.

A form of weir, particularly suitable for flow measurement, is the “notch”, so called because it comprises a sharp edged notch cut out of a metal plate. The cut out may of course be of any shape, but rectangular and V-shaped notches, as illustrated in Fig. 2.1, are generally preferred.

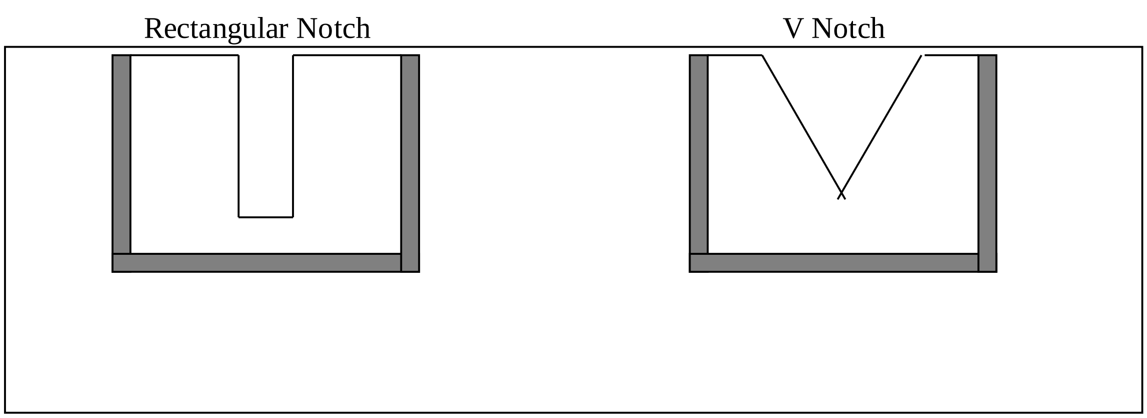


Fig. 2.1 Front view of rectangular and V notches set in a channel

The details of construction of the equipment vary somewhat between the gravimetric and volumetric types of hydraulic bench. In the gravimetric bench, the SS 304 open channel is used, the notches being screwed into the support in the walls. In the volumetric bench, the channel is moulded as part of the bench top rather than being supplied as a separate item of equipment. Water is admitted from the bench supply to the enlarged end of the channel, then over the weir into a collecting hopper, from which it is led to the weigh tank. The upstream level of the notch under test, is measured by a point gauge which is attached to the wall of the channel.

Theory of Flow over Sharp Edged Weirs:

Fig. 2.2 shows the essential features of flow over rectangular and V notches. The approaching flow is assumed to be uniform, with total head H measured above the crest level of the notch. If the cross-sectional area of the channel is much larger than that of the area of flow over the notch, then the velocity head in the approaching flow will be negligible, and the water level there will represent the total head H. Throughout the undisturbed flow upstream, conditions are effectively hydrostatic, the total head H at a typical point M being given by

$$H = \frac{p_M}{w + z_M}$$

(3.1)

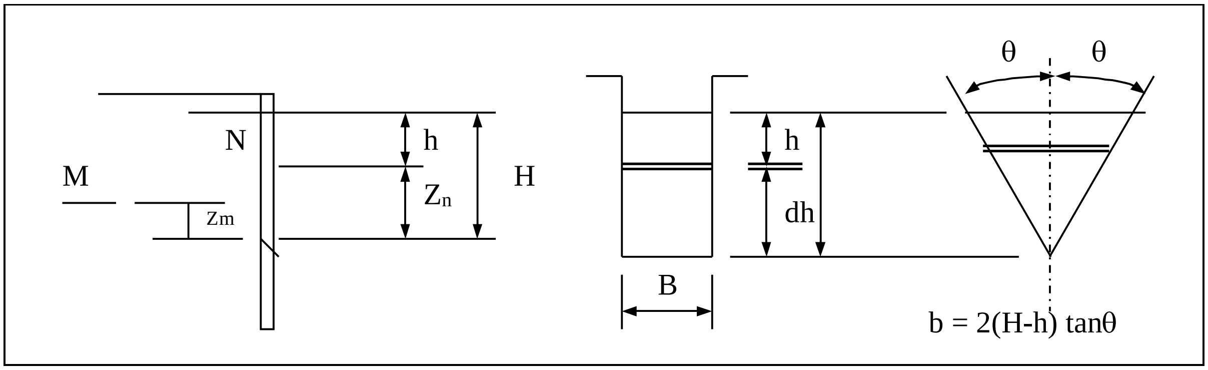


Fig. 3.2 Diagrammatic sketch of flow over sharp edged notches

Consider a typical streamline of the flow, from the typical point M in the upstream section to the point N in the plane of the notch. In the absence of any loss of total head, according to Bernoulli's equation.

$$H = \frac{p_M}{w} + z_M = \frac{u_N^2}{2g} + \frac{p_N}{w} + z_N$$

(3.2)

The static pressure P_N at N is now assumed to be atmospheric, viz. $p_N = 0$, so that

$$H = \frac{u_n^2}{2g} + z_N$$

Using the substitution

$$H - z_N = h$$

Where h is the depth of the point N below the undisturbed surface level, as shown in Fig 2.2, we obtain

$$u_N = \sqrt{2gh}$$

(3.3)

The flow velocity a N is thus seen to be that which would be attained by a particle falling freely from the level of the undisturbed upstream surface. The discharge rate over any notch may now be found by integration. If b is the width of the notch at depth h below the upstream surface level, the for an element of height δh, the element of area δA is

$$\delta A = b \ \delta h$$

so the element of discharge δQ is

$$\delta Q = u_N \delta A = \sqrt{2gh} \ b\delta h$$

The total flow rate Q, obtained by integration form zero to H, is then

$$Q = \int_0^H \sqrt{2gh} \ b \ dh$$

(3.4)

This result applies to a notch of any shape, and may easily be evaluated for the rectangular and V shapes under discussions.

For the rectangular notch of breadth B, Equation (2.4) becomes

$$Q = B\int_0^H \sqrt{2gh} \ dh$$

Which integrates immediately to give

$$Q = \frac{2}{3}\sqrt{2g} \ B \ H^{3/2}$$

(3.5)

For the V notch of angle 2θ , the width b is given by

$$b = 2(H - h)\tan\theta$$

The total flow rate is therefore given by Equation (2.4) as

$$Q = \int_0^H \sqrt{2gh} \, 2(H - h) \tan\theta \ dh$$

The result of integration is

$$Q = \frac{8}{15}\sqrt{2g} \tan\theta \ H^{5/2}$$

(3.6)

Each of the theoretical results given in Equations (2.5) and (2.6) ignore the contraction of the flow as it passes through the notch. This may be seen clearly on the apparatus, both in the vertical plane, where the upper surface is drawn downwards over the notch and the lower surface springs from the crest in an upward direction, and in the horizontal plane, where the water springs form the edges of the notch in a curve which reduces the width of the stream. The contraction is similar to that observed at a sharp edged orifice, and has the same effect of reducing the discharge rate. We therefore rewrite the equations as

$$Q = C_d \frac{2}{3} \sqrt{2g} \; B \; H^{3/2}$$

(3.7)

for the rectangular notch, and

$$Q = C_d \frac{8}{15} \sqrt{2g} \tan\theta \; H^{5/2}$$

(2.8)

for the V notch.

In these, C_d is a dimensionless coefficient of discharge of the notch, which is not necessarily independent of H, and which is to be determined by experiment.

Experimental Procedure:

The apparatus in first connected to the supply. The point gauge is then read or set to zero when it is exactly at the level of the crest of the notch. This is done by admitting water from the bench supply until the channel is filled with water and start spilling over the notch to the outlet. Water supply is stopped by closing the bench supply valve with pump in running condition (with by pass valve fully open) and allowing the water above the crest height to spill over the notch. The point gauge is also brought exactly to the water surface, and the calibrated dial is read.

Sets of measurements of discharge rate and head are taken for each weir in turn, the flow being regulated by bench supply valve. It is recommended that the first reading be taken at maximum flow rate, and subsequent values with roughly equal decrements in head. Readings should be discontinued when the level has fallen to the condition at which the stream ceases to spring clear of the notch plate. As it takes a little time for the water level to stabilise after changing the flow rate, care should be taken to ensure that conditions have settled completely before starting to measure the discharge. About 5 experimental points for each notch should be sufficient.

The width B of the rectangular notch and the angle θ of the V notch should be recorded.

Results and Calculations:

Width of rectangular notch = mm
V-Notch 1:
Width across top of V notch = mm
Depth of V notch = mm

Angle of V notch θ = tan θ = $\frac{Width\ across\ top\ of\ V\ notch}{2 * Depth\ of\ V\ notch}$ = = °

Table 2.1 presents results of measurements on the rectangular notch. The first 3 columns show readings obtained in the laboratory. The head H shown in the fourth column is obtained by subtracting the initial gauge reading; in the second line of the table, for example,

$$H = 62.61 - 3.93 = 58.68 \text{ mm}$$

The discharge rate Q is obtained from the time t required to collect quantity of 30 kg for example,

$$Q = 30/ 39.4 = 0.761 \text{ kg/ s} = 0.761 \text{ l/s} = 7.61 * 10^{-4} \text{ m}^3\text{/ sec}$$

So,

$$10^4 *Q = 7.61 \text{ m}^3\text{/ s}$$

Since we expect, from equation (2.4), that Q will vary as $H^{3/2}$, values of $H^{3/2}$ are also tabulated. For example, in the second line of the table,

$$H = 58.68 \text{ mm} = 0.05868 \text{ m}$$

Hence,

$$H^{3/2} = 0.01422 \text{ m}^{3/2}$$

$$10^2 * H^{3/2} = 1.422 \text{ m}^{3/2}$$

Table 2.2 shows corresponding results for the V notch, except that the exponent of H is now 5/2, obtained from equation 2.8.

The last column of Table 2.1 shows individual values of discharge coefficient C_d, calculated from a rearranged form of equation (2.4):

$$C_d = \frac{Q}{\left(2/3\right) B\sqrt{2g}H^{3/2}}$$

Plot discharge rate 10^4*Q as a function of H and of $H^{3/2}$. This later graph is seen to be a straight line through the origin, so providing experimental confirmation of the theoretical prediction of eqn. (2.4).

The slope of this straight line may be used to calculate the mean value of C_d as follows. The slope is

$$\frac{10^4 * Q}{10^2 * H^{3/2}} =$$

So that,

$$\frac{Q}{H^{3/2}} =$$

Substituting numerical values in equation given above, we obtain

$$C_d =$$

The results obtained from the V notch are treated in the same manner.

Discussion of Results:

The results show that the discharge over the rectangular notch is well represented by the equation

$$Q = \frac{2}{3}\sqrt{2g}\; B\; H^{3/2}$$

in which the value of C_d is

$$C_d \quad =$$

Over the range of experiment. The expression $2/3\sqrt{2g} * B*H^{3/2}$ which appears in the equation is the discharge calculated by neglecting the losses and the contraction of the jet as it passes over the notch. C_d is a dimensionless discharge coefficient which takes account of these effects. Similarly for the V notch, the discharge is well represented by

$$Q = C_d \frac{8}{15}\sqrt{2g}\; \tan\theta\; H^{5/2}$$

Where the discharge coefficient C_d again has the value

$$C_d \quad =$$

These values are typical of sharp edged notches, and indeed, of flow through other forms of sharp edged contraction such as orifice plates. Other set of data, obtained under different experimental conditions, may well lead to somewhat different values of C_d. Figures in the range from 0.60 to 0.65 are generally to be expected.

SAMPLE CALCULATION:

OBSERVATION:

Triangular Notch:

Angle of V - notch θ = 60 º

Area of the measuring tank = 0.16 m^2

Sr. No	Initial reading (h1) cm	Final reading (h2) cm	Volume V CM^3	Time (sec)	Q act = V/t Cm^3/s	Log Q
1	7	15	12	60	213.33	2.32
2	5	18.5	21	60	360	2.55
3	6.5	23	26	60	440	2.64
4	5.5	15.5	16	30	533.33	2.72

Sr. No	Pointer reading (H_1) cm	Pointer reading (H_2) cm	H = $H_2 - H_1$	$H^{5/2}$	*Log H*	C_d
1	12.2	15.7	3.5	22.9	0.54	0.69
2	12.2	15.7	4.5	42.9	0.65	0.64
3	12.2	15.7	5	55.9	0.69	0.58
4	12.2	15.7	5.3	64.6	0.72	0.61

Rate of Flow given by:

$$Q = C_d \frac{8}{15} \sqrt{2g} \tan\theta \ H^{5/2}$$

So, Discharge co efficient:

C_d = Q /[(8/15) * (2*g) $^{1/2}$ * tan θ * $H^{5/2}$]

FOR READING 1:

C_d = Q /[(8/15) * (2*g) $^{1/2}$ * tan θ * $H^{5/2}$]

= 213.33 / (0.53*44.29*0.57*22.9)

= 0.69

OBSERVATION:

Triangular Notch:

Angle of V - notch θ =

Area of the measuring tank =

Sr. No	**Initial reading (h1) cm**	**Final reading (h2) cm**	**Volume V CM^3**	**Time (sec)**	**Q act = V/t Cm^3/s**	**Log Q**
1						
2						
3						
4						

Sr. No	**Pointer reading (H_1) cm**	**Pointer reading (H_2) Cm**	**H = $H_2 - H_1$**	**$H^{5/2}$**	***Log H***	**C_d**
1						
2						
3						
4						

Rate of Flow given By:

$$\mathbf{Q} = C_d \frac{8}{15} \sqrt{2g} \tan\theta \; H^{5/2}$$

So, Discharge co efficient:

$C_d = Q /[(8/15) * (2*g)^{1/2} * \tan\theta * H^{5/2}]$

FOR READING 4:

$C_d = Q /[(8/15) * (2*g)^{1/2} * \tan\theta * H^{5/2}]$

=

4. Friction Losses in Pipe Fittings

INTRODUCTION:

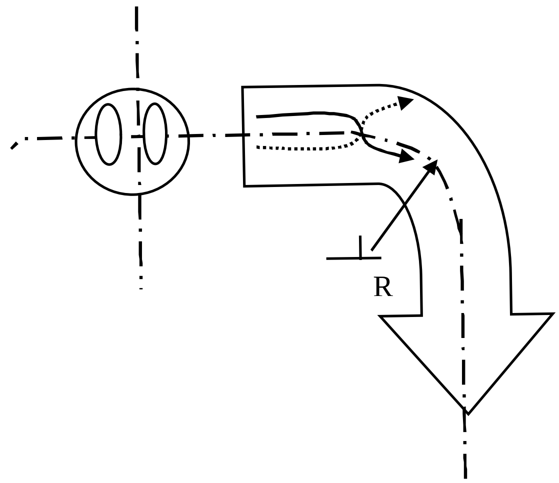
In Hydraulic Engineering practice, it is frequently necessary to estimate the head loss incurred by a fluid as it flows along a pipeline. For example, it may be desired to predict the rate of flow along a proposed pipe connecting two reservoirs at different levels. Or it may be necessary to calculate what additional head would be required to double the rate of flow along an existing pipeline.

Loss of head is incurred by fluid mixing, which occurs at fittings such as bends or valves, and by frictional resistance at the pipe wall. Where there are numerous fittings and the pipe is short, the major part of the head loss will be due to the local mixing near the fittings. For a long pipeline, on the other hand, skin friction at the pipe wall will predominate. In the experiment described below, we investigate the frictional resistance to flow along a long straight pipe with smooth walls.

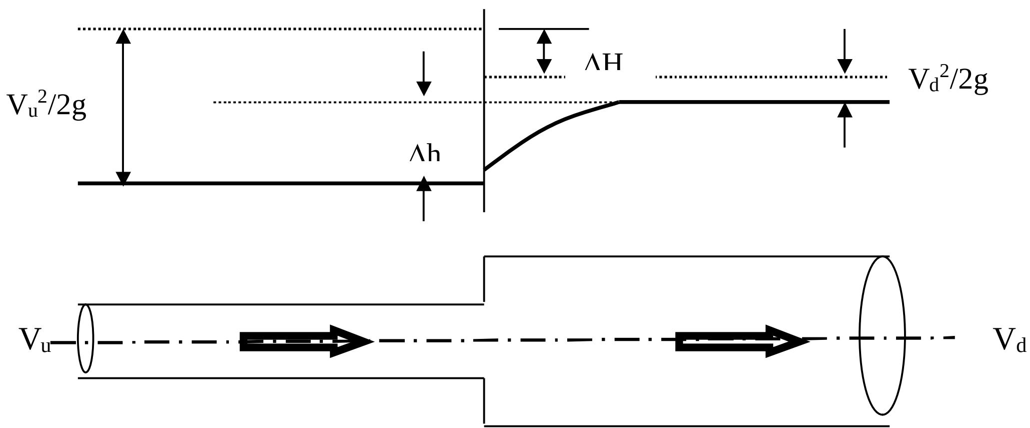
⇒ **Flow in Bend:**

Figure illustrates flow round a 90^0 bend which has a constant circular cross section of diameter D. the radius of the bend is R, measured to the center line. A radial as it passes round the bend causes the curvature of the flow as it passes round the bend is caused by a radial gradient of piezometric head, so that the piezometric is lower at the inner surface of the pipe than its outer surface. As the flow leaves the bend, these heads starts to equalize as the flow loses its curvature, so that the piezometric head begins to rise along the inner surface. This rise causes the flow to separate, so generating mixing losses in the subsequent turbulent reattachment process. Additionally, the radial gradient of piezometric head sets up a secondary cross flow in the form of a pair of vortices, having outward directed velocity components near the pipe center, and inward components near the pipe walls. When superimposed on the general streaming flow, the result is a double spiral section, which persists for a considerable distance in the downstream flow, and which generates further losses that are attributable to the bend.

Clearly, the value of the loss co-efficient will be a function of the geometric ratio R/D; as this ratio increases, making the bend less sharp, we would expect the value of K to fall. The smallest possible valve of R/D is 0.5, for which the bend has a sharp inner corner. For this case, the value of K is usually about 1.4. As R/D increases up to 2 or 3. There is also a slight dependence on Reynolds's number N_{Re}.



⇒ **Flow in Sudden Enlargement:**



The flow in a sudden separate at the exit from the smaller pipe, forming a jet which diffuses into the larger bore, and which reattaches to the wall some distance downstream. The vigorous turbulent mixing, resulting from the separation and reattachment of flow, causes a **loss of total head**. The **piezometric head** in the emerging jet, however, starts at the same value as in the pipe immediately upstream, and increases **through the mixing region**, so rising across the enlargement. These changes in total and piezometric head, neglecting the effect of friction gradient, are illustrated in the figure. Assuming that the piezometric pressure on the face of the enlargement to be equal to that in the emerging jet, and that the momentums flux is conserved; the loss of total head may be shown to be:

$$\frac{(V_u - V_d)^2}{2g} == \Delta H$$

The corresponding rise in piezometric head is

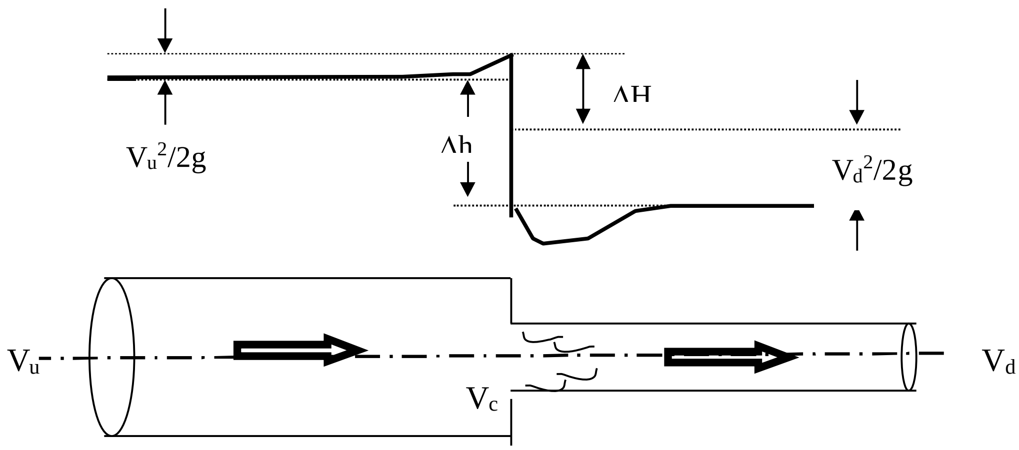
$$\frac{2V_d(V_u - V_d)}{2g} == \Delta h$$

The loss coefficient K in this case best related to the upstream velocity V_u so that

$$K_e == \frac{(V_u - V_d)^2 / 2g}{V_u^2 / 2g} == \left[1 - \frac{V_d}{V_u}\right]^2 == \left[1 - \frac{A_d}{A_u}\right]^2$$

This indicates that K increases from zero when A_u / A_d = 1.0 (the case when there is no enlargement), to 1.0 when A_u / A_d falls to zero.

⇒ **Flow in sudden contraction:**



The flow in a sudden separates at the exit from the edge where the face of contraction leds into the smaller pipe, forming a jet which converges into the contracted section of cross sectional area A_c. beyond this contracted section there is a region of turbulent mixing, in which the jet diffuses and reattaches to the wall of the downstream pipe. The losses occur almost entirely in the process of turbulent diffusion and reattachment. The losses are therefore expected to be those due to an enlargement from contracted area A_c to the downstream pipe area A_d . The expected loss of total head in contraction is:

$$\frac{(V_c - V_d)^2}{2g} == \Delta H$$

The corresponding rise in piezometric head is

$$\frac{2V_d(V_c - V_d)}{2g} == \Delta h$$

The loss coefficient K in this case best related to the upstream velocity V_c so that

$$K_c == \frac{(V_d - V_c)^2 / 2g}{V_c^{\,2} / 2g} == \left[\frac{V_d}{V_c} - 1\right]^2 == \left[\frac{A_d}{A_c} - 1\right]^2$$

This indicates that K increases from zero when A_d / A_c = 1.0 (the case when there is no contraction), to 1.0 when A_d / A_c falls to zero. If however, the contraction is very sever, viz. A_d / A_c $\rightarrow$ 0, then the upstream pipe tends to an infinite reservoir in comparison with the downstream one. We might then form a large reservoir through an orifice of area A_d. For such an orifice the contraction coefficient has the value of approximately, so that

$$\frac{A_d}{A_c} = \frac{1}{0.6} = 1.667 \qquad\qquad K = 1.667$$

It might therefore be expected that L would rise from zero when the pipe area ratio A_d / A_u = 1 to a value of about 0.44 as the ratio A_d / A_u falls towards zero.

⇒ ESTIMATING FRICTION LOSSES:

Fluid flow is always accompanied by friction and an accompanying loss of energy. This results in a pressure drop in the direction of flow (the pressure downstream will be lower than the pressure upstream).

Darcy's Equation:

$h_f = f\, L\, v^2 / 2\, D\, g_c$

The pressure drop, h_f (J/kg), is proportional to the square of the velocity, v (m/s²), the length of the pipe, L (m), and inversely proportional to the pipe diameter, D (m); g_c = 1 kg-m/s²/N.

In the laminar flow regime, the friction factor, f, can be found from:

$f = 64 / N_{Re}$

The Reynolds’s Number, $N_{Re} < 2000$, for laminar flow.

In the turbulent region, the friction factor, f, is a function of the Reynolds’s Number, N_{Re}, and the Relative Roughness (e/D). The relative roughness parameter is dimensionless ratio of the absolute pipe roughness, e, and the pipe diameter, D. The value of f for a given set of conditions is generally found from a Moody Friction Chart.

Alternatively, f, for turbulent flow, can be determined us the Sacham equation (± 1%):

$f = \{- 2\log\{(e/D)/3.7 - (5.02/N_{Re})\log[(e/D)/3.7 + 14.5/N_{Re}]\}\}^{-2}$

The friction losses that result from turbulent flow through various types of valves and fitting are summarized below:

TYPE OF FITTING	FRICTION LOSS, NUMBER OF VELOCITY HEADS, K_F	FRICTION LOSS, EQUIVALENT LENGTH OF STRAIGHT PIPE IN PIPE DIAMETERS, L_E/D
Elbow 45º	0.35	17
Elbow 90º	0.75	35
Tee	1	50
Return Bend	1.5	75
Coupling	0.04	2
Union	0.04	2
Gate Valve, wide open	0.17	9
Gate Valve, 1/2 open	4.5	225
Globe Valve, wide open	6.0	300
Globe Valve, 1/2 open	9.5	475
Angle Valve	2.0	100
Check Valve, Ball	70.0	3500
Check Valve, Swing	2.0	100
Water Meter, Disk	7.0	350

The friction losses that occur for laminar flow through various types of valves and fittings are summarized below:

TYPE OF FITTING	FRICTION LOSS, NUMBER OF VELOCITY HEADS, K_F VS. N_{RE}					
	50	100	200	400	1000	Turbulent
Elbow 90º	17	7	2.5	1.2	0.85	0.75
Tee	9	4.8	3.0	2.0	1.4	1.0
Globe Valve	28	22	17	14	10	6
Check Valve, Swing	55	17	9	5.8	3.2	2.0

You should also estimate the friction losses that occur due to sudden expansion and contractions.

EXPERIMENTAL SET-UP:

The equipment is mounted on a free - standing framework, which supports the test pipe work and instrumentation. The following typical pipefitting is incorporated for study: 90^0 large radius bend, sudden contraction, and sudden enlargement. All are instrumented with upstream and down stream pressure tapings. These tapings are connected to a bank of four manometer tubes, mounted on the acrylic board framework.

A gate valve is used to control the flow rate. A separate valve is instrumented with upstream and downstream pressure tapings that could be connected to the given manometer tubes.

The unit stands on the MS frame with legs. Water is supplied to the pipe inlet from the Supply tank, and discharged in to the measuring tank / supply tank.

Operating Manual

AIM:

1. To measure the losses in the devices related to flow rate and calculating loss coefficient related to velocity head.
2. To compare the pressure drop across each devices.

PROCEDURE:

- The diameter of the pipes and dimensions of the fittings are noted.
- The supply of the Supply tank is connected to the pipe work inlet. A further divider is fixed to the exit pipe, so that the discharge from the equipment flows into the measuring tank / supply tank of the bench.
- Connect all the manometer tapings to respective manometers with the help of rubber hose.
- The pump is then started, and the control valve at the exit is opened to allow water to circulate through the pipe work.
- To ensure that all air is expelled from the system, the air bleed valves at the top of the manometers are opened and the control valve should be closed sufficiently as to produce vigorous flow out of the air bleed valve, so ensuring that the system is thoroughly purged of air. When this is complete, the air vent valve should be closed.

- The apparatus is now ready for use. The exit valve is opened carefully, while the mercury levels are observed in the manometers tubes.
- When the maximum feasible flow rate is reached, differential manometers readings across each of the fittings are recorded, while the collection of a known quantity of water in the measuring tank of the bench is timed.
- These measurements are repeated at a number of rates of flow.

OBSERVATIONS:

Diameter of smaller bore pipe : D_1 = 16 mm
Diameter of larger bore pipe : D_2 = 26 mm
Radius of centerline of 90^0 bends : R_b = 55 mm
Manometer Range =
Number of Manometer =

Piezometric Head Losses at Various Rates of Flow

Sr. No.	Volume Collected (m^3)	Time (Sec.)	Flow Rate Q (m^3/S)	Difference in Manometer Limbs Δh (cmHg)				
				Valve1 – 2	Bend 3 - 4	Enlrg't 5 – 6	Cont'n 7 - 8	Bend 9-10
1								
2								

fitting	Δh	Δhf = 12.6* Δh m	Velocity Head m	Piezometric Head m	Total Head m	K
Sudden Contraction						
Sudden Enlargement						
90^0 Bend						

⇒ **CALCULATION:**

⇒ RESULT:

1. **The result for the elbow and bend show that the loss coefficient K falls as the radius of the band is increased.**
2. For Enlargement the measured value is significantly higher.

⇒ **CONCLUSION:**

OBSERVATIONS:

Diameter of smaller bore pipe : D_1 = 16 mm
Diameter of larger bore pipe : D_2 = 26 mm
Radius of centerline of 90^0 bends : R_b = 55 mm
Manometer Range = mm
Number of Manometer =

Piezometric Head Losses at Various Rates of Flow

Sr. No.	Volume Collected (m^3)	Time (Sec.)	Flow Rate Q (m^3/S)	Difference in Manometer Limbs Δh (cmHg)				
				Valve1 – 2	Bend 3 - 4	Enlrg't 5 – 6	Cont'n 7 - 8	Bend 9-10
1								
2								

Loss in Expansion:

X1= CM
X2= CM
ΔX = X2 – X1 = = cm = m
VOLUME COLLECTED:
V =
VOLUMETRIC FLOWRATE
Q =
=

CROSS SECTION AREA OF THE PIPE:

$A1 = \pi/4\ (DI)^2$

=

A2=

VELOCITY:

V1= Q/A1

$= 6.15*10^{-4}/0.00020096$

= 3.060 m/s

V2= Q/A2

=

VELOCITY HEAD:

$V1^2/2g$ = (

$V2^2/2g$ =

REYNOLDS NUMBER :

Re1= ρ V D/μ

=

Re2= ρ V D/μ

=

Re	10000	15000	20000	25000	30000	35000
F	0.00773	0.00696	0.00648	0.00614	0.00588	0.00567

FRICTION FACTOR:

f1 =

f2 =

$\Delta hf1 = 4 * f1 * L1/D1 * V1^2/2g$

$\Delta hf2 = 4 * f2 * L1/D1 * V1^2/2g$

=

$\Delta hf = \Delta hf1 + \Delta hf2$

=

Manometer height difference:

h1= cm

h2= cm

Δh= cm

Piezometric head loss (Δh) m

= 12.6 * Δh

=

Piezometric head difference (Δh) m

= Piezometric head loss (Δh) m – Friction head loss Δhf

=

Total head Loss (ΔH) m

= $\Delta h + V1^2/2g - V2^2/2g$

Loss coefficient K

= Total head loss / velocity head

=

Loss in Contraction:

X1= CM

X2= CM

ΔX = X2 – X1 = = cm =

VOLUME COLLECTED

V = m^3

VOLUMETRIC FLOWRATE

Q = m^3/s

CROSS SECTION AREA OF THE PIPE:

A1= $\pi/4\ (DI)^2$

= $3.14 * (0.016)^2/4$

= $0.00020096\ m^3$

A2= $\pi/4\ (DI)^2$

= $3.14 * (0.026)^2/4$

= $0.00053066\ m^3$

VELOCITY:

V1= Q/A1

=

V2= Q/A2

=

VELOCITY HEAD:

$V1^2/2g$ = (m

$V2^2/2g$ = m

REYNOLDS NUMBER :

Re1= ρ V D/μ

=

Re2= ρ V D/μ

=

Re	10000	15000	20000	25000	30000	35000
F	0.00773	0.00696	0.00648	0.00614	0.00588	0.00567

FRICTION FACTOR:

f1 = 0.00567

f2 = 0.00588

$\Delta hf1 = 4 * f1 * L1/D1 * V1^2/2g$

=

$\Delta hf2 = 4 * f2 * L1/D1 * V1^2/2g$

$\Delta hf = \Delta hf1 + \Delta hf2$

Manometer height difference:

h1= cm

h2= cm

Δh= cm

Piezometric head loss (Δh) m

= 12.6 * Δh

=

Piezometric head difference (Δh) m

= Piezometric head loss (Δh) m – Friction head loss Δhf

= 0.7308 – 0.049

= 0.6818 m

Total head Loss (ΔH) m

$= \Delta h + V1^2/2g - V2^2/2g$

=

Loss coefficient K

= Total head loss / velocity head

=

Loss in Band:

X1= CM

X2= CM

$\Delta X = X2 - X1 =$

VOLUME COLLECTED:

$V = 0.4 * 0.4 * 0.05 = 0.008\ m^3$

VOLUMETRIC FLOWRATE

Q **= 0.008/13**

= $6.15*10^{-4}\ m^3/s$

CROSS SECTION AREA OF THE PIPE:

$A1 = \pi/4\ (DI)^2$

=

VELOCITY:

V1= Q/A1

=

VELOCITY HEAD:

$V1^2/2g$ =

REYNOLDS NUMBER :

Re1= ρ V D/μ

=

f1 =

L1/D1 =

Δhf = 4 * f1 * (L1/D1) * $V1^2/2g$

=

Manometer height difference:

h1= cm

h2= cm

Δh= cm

Piezometric head loss (Δh) m

= 12.6 * Δh

=

Piezometric head difference (Δh) m

= Piezometric head loss (Δh) m – Friction head loss Δhf

=

Total head Loss (ΔH) m

= Δh + $V1^2/2g$

=

Loss coefficient K

= Total head loss / velocity head

=

Viscosity Kg/ m s	**μ**	0.001
Density of Water kg/m^3	**ρ**	1000

Fitting	**Δh m**	**Δhf = 12.6* Δh**	**Velocity Head m**	**Piezometric Head loss m**	**Total Head m**	**K**

		m				
Sudden Contraction						
Sudden Enlargement						
90^0 Bend						

⇒ RESULT:

The result for the elbow and bend show that the loss coefficient K falls as the radius of the band is increased.

For Enlargement the measured value is significantly high.

3. FRICTION LOSS ALONG A PIPE

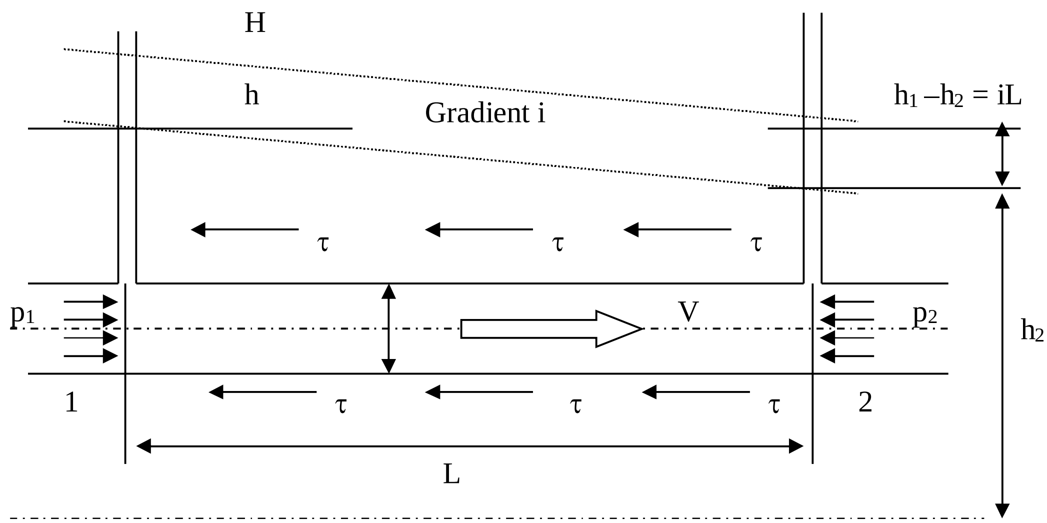
Introduction:

In hydraulic engineering practice, it is frequently necessary to estimate the head loss incurred by a fluid as it flows along a pipeline. For example, it may be desired to predict the rate of flow along a proposed pipe connecting two reservoirs at different levels. Or it may be necessary to calculate what additional head would be required to double the rate of flow along an existing pipeline.

Loss of head is incurred by fluid mixing which occurs at fittings such as bends or valves, and by frictional resistance at the pipe wall. Where there are numerous fittings and the pipe is short, the major part of the head loss will be due to the local mixing near the fittings. For a long pipeline, on the other hand, skin friction at the pipe wall will predominate. In the experiment described below, we investigate the frictional resistance to flow along a long straight pipe with smooth walls.

Friction Loss in Laminar and Turbulent Pipe Flow:

Figure 3.1 illustrates flow along a length of straight uniform pipe of diameter D. All fittings such as valves or bends are sufficiently remote as to ensure that any disturbances due to them have died away so that the distribution of velocity across the pipe cross section does not change along the length of pipe under consideration.



Such a flow is said to be "fully developed". The shear stress τ at the wall, which is uniform around the perimeter and along the length, produces resistance to the flow. The presure head h therefore falls at a uniform rate along the length, as shown in the figure 3.1. Since the velocity head is constant along the length of the pipe, the total head H also falls at the same rate. The slope of pressure line is also frequently called the "hydraulic gradient", and is denoted by symbol **i**:

$$i = -\frac{dh}{dl} = -\frac{dH}{dl}$$

(3.1)

(The minus sign are due to the fact that the head decreases in the direction of increasing l, which is measured positive in the same sense as the velocity V. The resulting value of I is then positive). Over the length L between sections 1 and 2, the fall in pressure head is

$$h_1 - h_2 = iL$$

(3.2)

Expressed in terms of pressures p_1 and p_2 at sections 1 and 2:

$$p_1 - p_2 = wiL = \rho giL$$

(3.3)

in which w is the specific weight and ρ is the density of water.

There is a simple relationship between wall shear stress τ and hydraulic gradient i. The pressures p_1 and p_2 acting on the two ends of the length L of pipe produce a net force. This force, in the direction of flow, is

$$(p_1 - p_2)\ A$$

In which A is the cross sectional area of the pipe. This is opposed by an equal and opposite force generated by the shear stress τ acting uniformly over the surface of the pipe wall. The area of pipe wall is P*L, where P is the perimeter of the cross section, so the force due to shear stress is

$$\tau * P * L$$

Equating these forces:

$$(p_1 - p_2)\ A = \tau * P * L$$

This reduces, by the use of equation (3.3), to

$$\tau = \frac{A}{P} \rho g i \qquad \textbf{(3.4)}$$

Now expressing A and P in terms of pipe diameter D, namely $A = \pi D^2/4$ and $P = \pi D$ so that $(A/P) = D/4$, we obtain the result:

$$\tau = \left(\frac{D}{4}\right)\rho g i \tag{3.5}$$

We may reasonably expect that τ would increase in some way with increasing rate of flow. The relationship is not a simple one, and to understand it we must learn something about the nature of the motion, first described by Osborne Reynolds in 1883. By observing the behavior of a filament of dye introduced into the flow along a glass tube, he demonstrated the existence of two different types of motion. At low velocities, the filament appeared as a straight line passing down the whole length of the tube, indicating smooth or laminar flow. As the velocity was gradually increased in small steps, he observed that the filament, after passing a little way along the tube, mixed suddenly with the surrounding water, indicating a change to turbulent motion. Similarly, if the velocities were decreased in small steps, a transition from turbulent to laminar motion suddenly occurred. Experiments with pipes of different diameters and with water at various temperatures led Reynolds to conclude that the parameter which determines whether the flow shall be laminar or turbulent in any particular case is

$$\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{VD}{\nu} \tag{3.6}$$

In which

Re	=	Reynolds Number of the motion
ρ	=	Density of the fluid
V	=	Q/A denotes the mean velocity of flow, obtained by dividing the discharge rate Q by the cross sectional area A
μ	=	Coefficient of absolute viscosity of the fluid
ν	=	μ/ ρ denotes the coefficient of kinematics viscosity of the fluid

Note that the Reynolds number is dimensionless.

The motion will be laminar or turbulent according as to whether the value of Re is less than or greater than a certain critical value. Experiments made with increasing flow rates show that the critical value of Re for transition to turbulent flow depends on the degree of care taken to eliminate disturbances in the supply and along the pipe. On the other hand, experiments with decreasing flow rates show that transition to laminar flow takes place at a value of Re which is much less sensitive to initial disturbance. This lower value of Re is found experimentally to be about 2000. Below this, the pipe flow becomes laminar sufficiently downstream of any disturbance, no matter how severe.

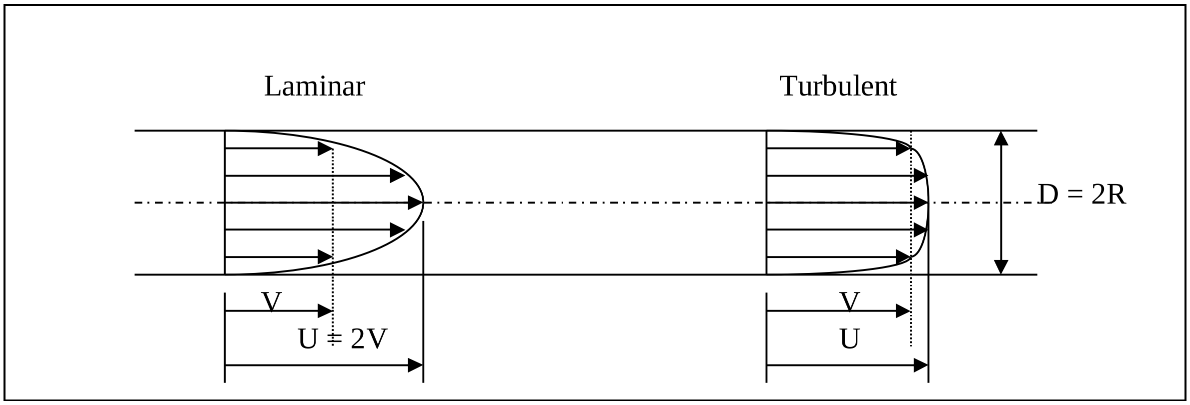


Fig 3.2 Velocity distributions in laminar and turbulent pipe flows

Figure 3.2 illustrates the difference between velocity profiles across the pipe cross sections in laminar and turbulent flow. In each case the velocity rises from zero at the wall to a maximum value U at the centre of the pipe. The mean velocity V is of course less than U in both cases.

In the case of laminar flow, the velocity profile is parabolic. The ratio U/V of centre line velocity to mean velocity is

$$\frac{U}{V} = 2 \tag{3.7}$$

and the velocity gradient at the wall is given by

$$\left(\frac{du}{dr}\right)_R = -\frac{4U}{D} = -\frac{8V}{D} \tag{3.8}$$

So that the wall shear stress τ due to fluid viscosity is

$$\tau = \frac{8\mu V}{D} \tag{3.9}$$

Substituting for τ in equation (3.5) from this equation leads to the result

$$i = \frac{32\nu V}{gD^2} \tag{3.10}$$

Which is the Poiseuille's equation.

In the case of turbulent flow, the velocity distribution is much flatter over most of the pipe cross section. As the Reynolds number increases, the profile becomes increasingly flat, the ratio of maximum to mean velocity reducing slightly. Typically, U/V falls from about 1.24 to about 1.12 as Re increases from 10^4 to 10^7.

Because of the turbulent nature of the flow, it is not possible now to find a simple expression for the wall shear stress, so the value has to be found experimentally. When considering such experimental results, we might reasonably relate the wall shear stress τ to the mean velocity pressure $\frac{1}{2}\rho V^2$. So a dimensionless **friction factor f** could be defined by

$$\tau = f \tfrac{1}{2} \rho V^2$$

(3.11)

The hydraulic gradient may now be expressed in terms of f by use of equation (3.5), and the following result is readily obtained:

$$i = \frac{4f}{D}\frac{V^2}{2g}$$

(3.12)

Therefore, the $(h_1 - h_2)$ between sections 1 and 2 of a pipe of diameter D, along which the mean velocity is V, is seen from equation (3.2) to be given by

$$h_1 - h_2 = 4f\frac{L}{D}\frac{V^2}{2g}$$

(3.13)

Where L is the length of pipe run between the sections. This is frequently referred to as Darcy's equation.

The results of many experiments on turbulent flow along pipes with smooth walls have shown f to be a slowly decreasing function of Re. Various correlations of the experimental data have been proposed, one of which is

$$\frac{1}{\sqrt{f}} = 4\log\left(\mathrm{Re}\sqrt{f}\right) - 0.4$$

(3.14)

This expression, which is due to Prandtl, fits experimental results well in the range of Re from 10^4 to 10^7, although it does have the slight disadvantage that f is not given explicitly.

Another correlation, due to Blasius, is:

$$f = 0.079\,\mathrm{Re}^{-1/4}$$

(3.15)

This gives explicit values which are in agreement with those from the more complicated equation (3.14) to within about 2% over the limited range of Re from 10^4 to 10^5. Above 10^5, however, the Blasius equation diverges substantially from experiment.

We have seen that when the flow is turbulent it is necessary to resort to experiment to find f as function of Re. However, in the case of laminar flow, the value of f may be found theoretically from Poiseuille’s equation. Equating the expressions for I in equation (3.10) and (3.12):

$$\frac{32 \nu V}{g D^2} = \frac{4f}{D}\frac{V^2}{2g}$$

after reduction this gives the result

$$f = \frac{16}{\mathrm{Re}}$$

(3.16)

In summary, the hydraulic gradient i may conveniently be expressed in terms of a dimensionless wall friction factor f. This factor has the theoretical value f =16/ Re for laminar flow along a smooth walled pipe. There is no corresponding theoretical for turbulent flow, but good correlation of many experimental results on smooth walled pipes is given by equations such as (3.14) and (3.15).

Description of Apparatus:

The apparatus consist of three SS 304 pipes of different diameters and 1 m test section length. Water from a supply tank is led through a flexible hose to the entrance of a single tube along which the friction loss is measured. Manometer tapings are made at an upstream and downstream of the test section which lies 500 mm away from the pipe entry and exit. These clear lengths upstream and downstream of the test section are required to ensure that the results are not affected by the disturbances originating at the entrance or the exit of the pipe. The manometer tapings are connected to a manometer, which reads head loss in mm of mecury gauge.

The rate of flow along the pipe is controlled by bench supply valve, and may be measured by timing the collection of water in weighing tank.

Experimental Procedure:

The apparatus is set on the bench. The mercury manometer is then connected to the manometer by rubber tube to one of the three pipes provided also connect the flexible supply hose and return hose to the same pipe. The bench supply valve is then carefully opened and adjusted until there is a steady flow down the pipe, so that it provides a constant head to the pipe under test. Particular care should be taken to remove all air from manometer connections.

Collect the predetermined quantity of water in the weighing tank and measure the time required for the same. Also record the reading of head loss (i.e. head difference in manometer limbs). The diameter D of the tube under test, and the length L between the manometer tappings, should be noted.

Results and Calculations:

Length of pipe between manometer tapings L = 1000 mm
Temperature of Water = °C

Pipe 1:
Diameter of Pipe, D = 10 mm
Cross sectional area of pipe $\pi D^2/4$ A = mm^2 = m^2

Sr. No.		t (sec)	ΔH (mm)	V (m/s)	i	log i	10^3*f	logf	Re	log Re

Pipe 2:
Diameter of Pipe, D = 12.5 mm
Cross sectional area of pipe $\pi D^2/4$ A = mm^2 = m^2

Sr. No.		t (sec)	ΔH (mm)	V (m/s)	i	log i	10^3*f	logf	Re	log Re

Pipe 3:
Diameter of Pipe, D = 16 mm
Cross sectional area of pipe $\pi D^2/4$ A = mm^2 = m^2

Sr. No.	)	t (sec)	ΔH (mm)	V (m/s)	i	log i	10^3*f	logf	Re	log Re

θ °C	0	1	2	3	4	5	6	7	8	9
10										
20										
30										

Table of $10^6\nu$ (m^2/s) as a function of water temperature θ °C

Values of ν, which are needed to compute Reynolds numbers, may be obtained by interpolation from this table. Alternatively, they may be obtained from the empirical formula

$$10^6\nu = 1.0049 - 0.02476(\theta-20) + 0.00044(\theta-20)^2$$

Which fits experimentally measured values of ν very well over the range of θ from 15 °C to 30 °C.

Hydraulic Gradient i is given by

$$i = \frac{12.6 * \Delta H}{L}$$

and log i, used for graphical representation, is

log i =

To obtain the friction factor f, we first compute the velocity head as follows:

The flow rate Q is

$$Q = \frac{Qty}{t} = \qquad m^3/\ sec$$

So the velocity along the pipe is

$$V = \frac{Q}{A} = \qquad m/s$$

The velocity head is then

$$\frac{V^2}{2g} = \qquad m$$

Equation (3.12) may now be used to find f:

$$i = \frac{4f}{D}\frac{V^2}{2g}$$

so that

$$f = i\frac{D}{4}\frac{1}{V^2/2g}$$

then

log f = $\qquad\qquad$ =

Finally, Reynolds number Re is obtained from the definition

$$\mathrm{Re} = \frac{VD}{\upsilon}$$

in which ν is found by interpolation from the table, or from equation.

and

log Re = $\qquad\qquad$ =

plot log i and log f as a function of log Re.

4. LOSSES AT PIPE FITTINGS

Introduction

As described in Chapter 3, loss of head along a pipeline is incurred both by frictional resistance at the wall along the run of the pipe, and at fittings such as bends or valves. For long pipes with few fittings, the overall loss is dominated by wall friction. If, however, the pipe is short and there are numerous fittings, then the principal losses are those which are produced by disturbances caused by the fittings. In the experiment described below, we investigate losses at various fittings, typical of those which are used frequently in pipe systems.

Measurement of Loss of Total Head at a Fitting:

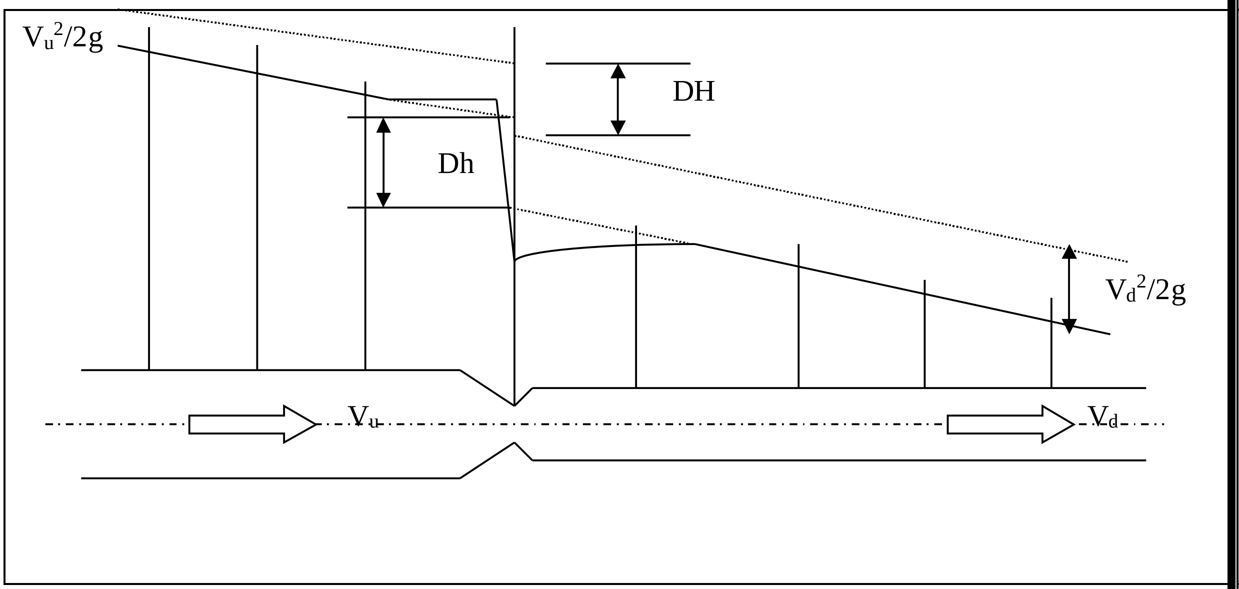


Fig. 4.1 Schematic representation of loss at a pipe fitting

Fig. 4.1 shows water flowing at speed V_u along a pipe of diameter D_u towards some pipe fitting such a bend or a valve, but shown for simplicity as a simple restriction in the cross section of the flow. Downstream of the fitting, the water flows along a pipe of some diameter D_d, along which the velocity of flow is V_d. The figure indicates the variation of piezometric head along the pipe run, as would be shown by numerous pressure tapings in the pipe wall. In the region of undisturbed flow, far upstream of the fitting, the distribution of velocity across the pipe remains unchanged from one cross section to another; this is the condition of fully developed pipe flow which is considered in Chapter 3. Over this region, the piezometric head falls with a uniform, mild gradient, as a result of constant friction at the pipe wall in the fully developed flow. Close to the fitting, however, there are sharp and substantial local disturbances to the piezometric head, caused by rapid changes in direction and speed as the water passes through the fitting. In the downstream region, these disturbances die away, and the line of piezometric head returns asymptotically to a slight linear gradient, as the velocity distribution gradually returns to the condition of fully developed pipe flow.

If the upstream and downstream lines of linear friction gradient are now extrapolated to the plane of fitting, a loss of piezometric head Δh due to the fitting is found. To establish the corresponding loss of total head ΔH it is necessary to introduce the velocity heads in the upstream and downstream runs of pipe. From figure 4.1 it is clear that

$$\Delta H = \Delta h + \frac{V_u^2}{2g} - \frac{V_d^2}{2g}$$

(4.1)

It is convenient to express this in terms of a dimensionless **loss coefficient K**, by dividing through by the velocity head in either the upstream or the downstream pipe (the choice depending on the context, as we shall see later). The result is

$$K = \frac{\Delta H}{V_u^2/2g} \quad or \quad \frac{\Delta H}{V_d^2/2g}$$

(4.2a)

For the case where $D_d = D_u$, the flow velocities in the upstream and downstream pipes are identical, so we may simplify the definition to

$$K = \frac{\Delta H}{V^2/2g} \quad or \quad \frac{\Delta h}{V^2/2g}$$

(4.2b)

Where V denotes the flow velocity in either the upstream or the downstream pipe run. (The velocity head used here is based simply on the mean flow velocity V). To obtain results of high accuracy, long sections of straight pipe, (of 60 pipe diameters or more), are needed to establish with certainty the relative positions of the linear sections of the piezometric lines. Such long upstream and downstream lengths are impracticable in a compact apparatus such as one described below. Instead, just two piezometric tapings are used, one placed upstream and the other downstream of the fitting, at sufficient distances as to avoid severe disturbances. These show the piezometric head loss **Δh' between the tapings**. An estimate is then made of the friction head loss Δh_f which would be incurred in fully developed flow along the run of pipe between the piezometric tapings. The piezometric head difference **Δh across the fitting** is then found by subtraction:

$$\Delta h = \Delta h' - \Delta h_f$$

(4.3)

Characteristics of Flow through Bends and at Changes in Diameter:

Figure 4.2(a) illustrates flow round a 90° bend which has a constant circular cross section of diameter D. The radius of the bend is R, measured to the center line. The curvature of the flow as it passes round the bend is caused by a radial gradient of piezometric head, so that the piezometric is lower at the inner surface of the pipe than its outer surface. As the flow leaves the bend, these heads start to equalise as the flow loses its curvature, so that the piezometric head begins to rise along the inner surface. This rise causes the flow to separate, so generating mixing losses in the subsequent turbulent reattachment process. Additionally, the radial gradient of piezometric head sets up a secondary cross flow in the form of a pair of vortices, having outward directed velocity components near the pipe centre, and inward components near the pipe walls. When superimposed on the general streaming flow, the result is a double spiral motion, which persists for a considerable distance in the downstream flow, and which generates further losses that are attributable to the bend.

Clearly, the value of the loss coefficient K will be a function of the geometric ratio R/D; as this ratio increases, making the bend less sharp, we would expect the value of K to fall. The smallest possible value of R/D is 0.5, for which the bend has a sharp inner corner. For this case, the value of K is usually about 1.4. As R/D increases, the value of K falls, reducing to values which may be as low as 0.2 as R/D increases up to 2 or 3. There is also a slight dependence on Reynolds Number Re.

Figure 4.2(b) shows the flow in a sudden enlargement. The flow separates at he exit from the smaller pipe, forming a jet which diffuses into the larger bore, and which reattaches to the wall some distance downstream. The vigorous turbulent mixing, resulting from the separation and reattachment of the flow, causes a **loss of total head**. The **piezometric head** in the emerging jet, however, starts at the same value as in the pipe immediately upstream, and **increases through the mixing region**, so rising across the enlargement. These changes in total and piezoetric head, neglecting the effect of friction gradient, are illustrated in the figure. Assuming that the piezometric pressure on the face of the enlargement to be equal to that in the emerging jet, and that the momentum flux is conserved, the loss of total head may be shown to be

$$\Delta H = \frac{(V_u - V_d)^2}{2g} \qquad \textbf{(4.4)}$$

The corresponding **rise in piezometric head is**

$$\Delta h = \frac{2V_d((V_u - V_d)}{2g} \qquad \textbf{(4.5)}$$

The loss coefficient K is in this case best related to the upstream velocity V_u so that

$$K = \frac{(V_u - V_d)^2 / 2g}{V_u^{\;2} / 2g} = \left[1 - \frac{V_d}{V_u}\right]^2 = \left[1 - \frac{A_d}{A_u}\right]^2 \qquad \textbf{(4.6)}$$

This indicates that K increases from zero when A_u/ A_d = 1.0(the case when there is no enlargement), to 1.0 when A_u/ A_d falls to zero.

Consider lastly the sudden contraction shown in figure 4.2(c). The flow separates from the edge where the face of contraction leads into the smaller pipe, forming a jet which converges to a contracted section of cross sectional area A_c. Beyond this contracted section there is a region of turbulent mixing, in which the jet diffuses and reattaches to the wall of the downstream pipe. The losses occur almost entirely in the process of turbulent diffusion and reattachment. The losses are therefore expected to be those due to an enlargement from the contracted area A_c to the downstream pipe area A_d. Following the result of equation (4.4), the expected loss of total head in contraction is

$$\Delta H = \frac{(V_c - V_d)^2}{2g}$$

(4.7)

The obvious choice of reference velocity in this case is V_d, so the loss coefficient K becomes

$$K = \left[\frac{V_d}{V_c} - 1\right]^2 = \left[\frac{A_d}{A_c} - 1\right]^2$$

(4.8)

Consider now the probable range of values of A_d/ A_c. If the value of the pipe contraction ratio is 1.0, that is if $A_d/A_u = 1.0$, than there is in effect no contraction and there will be no separation of flow, so $A_d/ A_c = 1.0$. Equation (4.8) then gives a zero value of K. If however, the contraction is very severe, viz. $A_d/ A_u \rightarrow 0$, then the upstream pipe tends to an infinite reservoir in comparison with the downstream one. We might then from a large reservoir through an orifice of area A_d. For such an orifice, the contraction coefficient has the value of approximately, so that

$$\frac{A_d}{A_c} = \frac{1}{0.6} = 1.667$$

Substituting this value in equation (4.7) gives

$$K = 0.44$$

It might therefore be expected that L would rise from zero when the pipe area ratio $A_d/A_u = 1$ to a value of about 0.44 as the ratio A_d/ A_u falls towards zero.

Description of Apparatus:

Several arrangements of apparatus are available, incorporating selection of fitting in various configurations. The particular equipment taken here has the advantage of portability. It may be operated from the hydraulic bench. It provides a run of pipe work, made up of components manufactured in Mild Steel, supported with legs. Water is supplied to the pipe inlet from the hydraulic bench, and is discharged at the exit to the measuring tank. In the run of the pipe there are the following pipe fittings:

- 90° miter bend
- 90° elbow bend
- 90° large radius bend
- Sudden enlargement in pipe diameter
- Sudden contraction in pipe diameter

Manometer tapings are provided in the pipe wall, at clear lengths of 4 pipe diameters, upstream and downstream of each of the fittings. The tapings are connected to an individual glass tube manometer. The flow rate through the equipment may be varied by adjusting the hydraulic bench supply valve and the valve near the pipe exit.

Experimental Procedure:

The diameters of the pipes and dimensions of the fittings are noted. The supply hose of the hydraulic bench is connected to the pipe work inlet. A further hose is fixed to the exit pipe, so that the discharge from the equipment flows into the measuring tank of the bench. Connect all the manometer tapings to respective manometers with the help of rubber hose. The pump is then started, and the control valve at the exit is opened to allow water to circulate through the pipe work.

To ensure that all air is expelled from the system, the air bleed valves at the top of the manometers are opened and the control valve at the exit is then partially closed, so that the pressure inside the pipe work drives water out through the air bleed valves. This flow of water will carry air bubbles along with it. The control valve should be closed sufficiently as to produce vigorous flow out of the air bleed valve, so ensuring that the system is thoroughly purged of air. When this is complete, the air vent valve should be closed.

The apparatus is now ready for use. The exit valve is opened carefully, while the water levels are observed in the manometer tubes. When the maximum feasible flow rate is reached, differential manometer readings across each of the fittings are recorded, while the collection of a known quantity of water in the measuring tank of the bench is timed. These measurements are repeated at a number of rates of flow.

Results and Discussions:

Dimensions of Pipes and Fittings

Diameter of smaller bore pipe $D_1 =$ 16 mm $A_1 =$ 201.062 mm^2 = 2.01062 * 10^{-4} m^2
Diameter of larger bore pipe $D_2 =$ 29 mm $A_2 =$ 660.52 mm^2 = 6.6052 * 10^{-4} m^2
Radius of center line of elbow $R_e =$ mm
Radius of center line of bend $R_b =$ mm
Length of straight pipe between manometer tapings and fitting $4D_1$ or $4D_2$

If the measured flow rate is Q m^3/ sec, then the velocities V_1 and V_2 along the pipes of cross sectional A_1 and A_2 m^2 areas are:

$$V_1 = Q/ A_1 \text{ m/s and } V_2 = Q/ A_2 \text{ m/s}$$

			Differential Piezometric Head Δh' (mm)				
Qty (kg)	**Time (s)**	**Q (kg/ s)**	**Mitre 1-2**	**Elbow 3-4**	**Enlrg't 5-6**	**Cont'n 7-8**	**Bend 9-10**

gives a typical set of results as recorded in the laboratory. Differential piezometric heads Δh' between manometer tapings are tabulated in sequence in the direction of flow, viz. Tapings 1 and 2 are upstream and downstream of the miter bend, 3 and 4 upstream and downstream of the elbow, and so on. Note that the reading for the enlargement is negative, showing increase of piezometric head at the enlargement.

					Loss of Total Head ΔH (mm)				
Q (kg/s)	**V_1 (m/s)**	**V_2 (m/s)**	**$V_1^2/2g$ (mm**	**$V_2^2/2g$ (mm)**	**Mitre 1-2**	**Elbow 3-4**	**Enlrg't 5-6**	**Cont'n 7-8**	**Bend 9-10**

Table 4.2 shows the head losses ΔH across each of the fittings, as computed from the measurements of Δh' in Table 4.2. The computations first use an estimate of the head loss Δh_f, due to friction between manometer tapings, to find the piezometric head loss Δh from equation (4.3). If the velocity downstream of the fitting is the same as that upstream, Equation (4.3) shows that the total head loss ΔH is the same as the piezometer head loss Δh. This is the case of miter, elbow and bend. If, however, there is a change in velocity from upstream to downstream, then equation (4.3) is used to compute total head loss ΔH from the piezonmetric head loss Δh.
The friction head loss is estimated by choosing a suitable value of friction factor f for fully developed flow along a smooth pipe. Several options are available, and the choice used here is Prandtl equation quoted in Chapter 3:

$$\frac{1}{\sqrt{f}} = 4 \log\left(\mathrm{Re}\sqrt{f}\right) - 0.4 \qquad \textbf{(4.14)}$$

Typical values derived from this equation, are presented in Table 4.3.

10^{-4} Re	1.0	1.5	2.0	2.5	3.0	3.5
10^3 f	7.73	6.96	6.48	6.14	5.88	5.67

Choosing the typical flow rate Q = 0.45 kg/ sec, and assuming the value of $\nu = 1.00 * 10^{-6}$ m^2/s for the coefficient of kinematic viscosity, then for the smaller bore pipe:

$$D_1 = 22.5 \text{ mm and } V_1 = 2.515 * 0.45 = 1.132 \text{ m/s}$$

so

$$Re_1 = V_1 \frac{D_1}{\nu} =$$

Similarly for larger bore pipe:

$$D_2 = 29.6 \text{ mm and } V_2 = 1.453 * 0.45 = 0.654 \text{ m/s}$$

so

$$Re_2 = V_2 \frac{D_2}{\nu} =$$

The values of friction factor at these two Reynolds number may be found from Table 4.3 by interpolation to be

$$f_1 = \text{ and } f_2 =$$

These are the values to be used to correct the observed differential heads Δh' in Table 4.1. For example, consider the mire bend. The pipe diameter is D_1, and the distance between the manometer tappings, measured along the pipe centerline, is given by

$$L_1 / D_1 =$$

Now Darcy's equation, presented as Equation (3.13) in Chapter 3, gives the frictional head loss Δh_f as

$$\Delta h_f = 4 f_1 \frac{L_1}{D_1} \frac{V_1^2}{2g}$$

Inserting numerical values:

$$\Delta h_f =$$

The piezometric head loss Δh across the mitre is then, according to Equation (4.3)

$$\Delta h = \Delta h' - \Delta h_f = \text{ mm}$$

Since there is no change in velocity from upstream to downstream of the mitre, this is also the loss of total head ΔH.

Similar calculations are made for the elbow and bend:

$$\frac{L_1}{D_1} = 8 + \frac{\pi R_1}{2 D_1}$$

This leads to

$$\Delta h_f = 0.257\ \frac{V_1^2}{2g}\ \text{for the elbow}$$

$$\Delta h_f = 0.313\ \frac{V_1^2}{2g}\ \text{for the bend}$$

Δh_f = 0.257 * 99.0 = 25.4 mm so ΔH = 113 – 25.4 = 87.6, say 88 mm for the elbow

Δh_f = 0.313 * 99.0 = 31.0 mm so ΔH = 62 – 31.0 = 87.6, say 31 mm for the bend

In the case of the enlargement, the sum of friction losses in the pipes of diameter D_1 upstream and of diameter D_2 downstream is

$$\Delta h_f = 4 f_1 \left(\frac{L_1}{D_1}\right)\frac{V_1^2}{2g} + 4 f_2 \left(\frac{L_2}{D_2}\right)\frac{V_2^2}{2g}$$

Inserting the numerical values:

or

Δh_f =

Then, for the enlargement:

Δh_f = mm

The change in piezometric head is then, from equation (4.3),

$\Delta h = \Delta h' - \Delta h_f$ = mm

To derive the change ΔH in total head, equation (4.1) is used:

$$\Delta H = \Delta h + \frac{V_1^2}{2g} + \frac{V_2^2}{2g} = \ \text{mm}$$

Similarly for the contraction, where Δh_f has the same value as for the enlargement. The computation is:

$\Delta h = \Delta h' - \Delta h_f$ = mm

$$\Delta H = \Delta h + \frac{V_2^2}{2g} - \frac{V_1^2}{2g} = \ \text{mm}$$

Derivation of Loss Coefficient K

Plot total head losses ΔH against velocity head for each of the fittings. In the case of miter, elbow and bend, the tube diameter is 22.5 mm, so the appropriate velocity head is obviously $V_1^2/2g$. For the enlargement and for the contraction, the relevant value is the velocity head in the pipe of smaller diameter, which again is $V_1^2/2g$. The slope of the line gives the value of K for the fitting.

Fitting	**K**
90° miter R/ D = 0.5	
90° elbow R/ D = 1.56	
90° bend R/ D = 3.07	
Enlargement D_u/D_d =	
Contraction D_u/D_d =	

Discussion of Results:

The result for the miter, elbow and bend show that the loss coefficient K falls substantially as the radius of the bend is increased.

For the enlargement, equation (4.6) provides a theoretical value of K. In this case, this theoretical value is

$$K = \left[1 - \frac{A_u}{A_d}\right]^2 = \left[1 - -\right]^2 =$$

The measured value is significantly higher. Perhaps the manometer tapping downstream of the enlargement is placed too close to allow the full recovery of piezometric pressure to take place. Moreover, the value of Δh_f in this case about one half that of ΔH. Therefore, if there is significant error in the computed effect of pipe friction, there will be a noticeable effect on the resulting value of K. For the contraction there is no theoretical value of K. However equation (4.8) may be used to calculate V_c / V_d from the measurements:

viz $\left[\frac{V_c}{V_d} - 1\right]^2 =$ from which $\frac{A_c}{A_d} = \frac{V_c}{V_d} =$

This is a plausible value for contraction coefficient of the jet at entry the contracted pipe, and it lies between the extreme value of about 0.6 and 1.0 discussed earlier.

5. IMPACT OF A JET

Introduction:

Water turbines are widely used throughout the world to generate power. In the type of water turbine referred to as a Pelton wheel, one or more water jets are directed tangentially on to vanes or buckets that are fastened to the rim of the turbine disc. The impact of the water on the vanes generates a torque on the wheel, causing it to rotate and to develop power. Although the concept is essentially simple, such turbines can generate considerable output at high efficiency. Powers is excess of 100 MW, and hydraulic efficiencies greater than 95%, are not uncommon. It may be noted that the Pelton wheel is best suited to conditions where the available head of water is great, and the flow rate is comparatively small. For example, with a head of 100 m and a flow rate of 1 m^3/s, a Pelton wheel running at some 250 rev/min could be used to develop about 900 kW. The same water power would be available if the head were only 10 m and the flow were $10 m^3/s$, but a different type of turbine would then be needed.

To predict the output of a Pelton wheel, and to determine its optimum rotational speed, we need to understand how the deflection of the jet generates a force on the buckets, and how the force is related to the rate of momentum flow in the jet. In this experiment, we measure the force generated by a jet of water striking a flat plate or a hemispherical cup, and compare the results with the computed momentum flow rate in the jet.

Description of Apparatus:

Water needed for generating the jet is supplied from the Hydraulic Bench through a vertical pipe terminating in a tapered nozzle. This produces a jet of water which impinges on a vane, in the form of flat plate or a hemispherical cup.

The nozzle and vane are contained within a transparent borosilicate glass cylinder, and at the base of the cylinder there is an outlet from which the flow is directed to the measuring tank of the bench. The vane is supported by a lever which houses the facility for applying jockey weight and which is restrained by a counter weight. The lever may be set to a balanced position (as indicated by a tally) by adjusting the counter balance position. Any force generated by impact of jet on the vane may now be measured by placing jockey weight at the end of the lever until the tally shows that it has been restored to its original balanced position.

Theory of the Experiment:

Consider how the equation of momentum applies to the case shown schematically in figure 5.1, which shows a jet of fluid impinging on a symmetrical vane.

Let the mass flow rate in the jet be $\dot{m}$. Imagine a control volume V, bounded by a control surface S which encloses the vane as shown. The velocity with which the jet enters the control volume is u_1, in the x-direction. The jet is deflected by its impingement on the vane, so that it leaves the control volume with velocity u_2, inclined at an angle β_2 to the x-direction. Now the pressure over the whole surface of the jet, apart from that part where it flows over the surface of the vane, is atmospheric. Therefore, neglecting the effect of gravity, the changed direction of the jet is due solely the force generated by pressure and shear stress at the vane's surface. If this force **on the jet** in the direction of x be denoted by F_j, the momentum equation in the x-direction is

$$F_j = \dot{m}(u_2 \cos \beta_2 - u_1)$$

(5.1)

The force F **on the vane** is equal and opposite to this, namely

$$F = \dot{m}(u_1 - u_2 \cos \beta_2)$$

(5.2)

For the case of a flat plate, $\beta_2 = 90°$, so that $\cos \beta_2 = 0$. It follows that

$$F = \dot{m}u_1$$

(5.3)

is the force on the flat plate, irrespective of the value of u_2.

For the case of a hemispherical cup, we assume that $\beta_2 = 180°$, so that $\cos \beta_2 = -1$, and

$$F = \dot{m}(u_1 + u_2)$$

(5.4)

If we neglect the effect of change of elevation on jet speed, and the loss of speed due to friction over the surface of the vane, then $u_1 = u_2$, so

$$F = 2\dot{m}u_1$$

(5.5)

is the maximum possible value of force on the hemispherical cup. This is just twice the force on the flate plate.

Returning now to Fig 5.1, the rate at which momentum is entering the control volume is $\dot{m}u_1$. We think of this as a rate of flow of momentum in the jet, and denote this by the symbol J, where

$$J = \dot{m}u_1$$

(5.6)

For the flat plate, therefore, we see form Equation (5.3) that

$$F = J$$

(5.7)

and for the hemispherical cup the maximum possible value of force is, from Equation (5.5)

$$F = 2J$$

(5.8)

In the SI system the units of $\dot{m}$ and u are

$$\dot{m}\,[kg/s] \text{ and } u\ [m/s]$$

In an equation such as (5.3), then the units of force F are

$$F[kg/s].[m/s] \text{or } [kg\ \ m/s^2] \text{ or } [N]$$

Experimental Procedure:

The lever is first brought to the balanced position (as indicated by the tally), without any jockey weight. Note the following dimensions: diameter of the nozzle, height of the vane above the tip of the nozzle when the lever is balanced, and distance from the pivot of the lever to the centre of the vane.

Water is then admitted through the bench supply valve, and the flow rate increased to the maximum. The force on the vane displaces the lever, which is then restored to its balanced position by adding the jockey weight at the other end of the lever. The mass flow rate is established by collection of water over a timed interval. Further observations are then made at a number of reducing flow rates. About eight readings should suffice.

The best way to set the conditions for reduced flow rate is to place the desired jockey weight, and then to adjust the flow control valve to bring the lever to the balanced position. The condition of balance is thereby found without touching the lever, which is much easier than finding the point of balance by adding the jockey weight.

The experiment should be run twice, first with the flat plate and then with the hemispherical cup.

Results and Calculations:

Diameter of Nozzle, D	= 10 mm
Cross sectional area of nozzle, A = πD^4	= $7.85 * 10^{-4}$ m^2
Height of vane above nozzle tip, s	= 90 mm = 0.09 m
Distance from the centre of vane to pivot of lever L	= 194 mm
Distance from the jockey weight to the pivot of lever, y	= 513 mm

When the jockey weight W is applied at a distance y mm from its zero weight, the force F on the vane which is required to restore balance is given by:

$$F * L = W * y$$

Assuming hypothetical value of Mass of jockey weight = 0.300 kg then weight of jockey weight W = Mg = 0.300 * 9.81 = 2.943 N

Inserting the value of W, namely 2.943 N, gives

$$F = (2.943 * 513)/194 = 7.7822\ N$$

The mass flow rate $\dot{m}$ in the jet is found by timing the collection of a known mass of water. The velocity of the jet as it leaves the nozzle is found from the volumetric flow rate Q and the cross-sectional area A of the nozzle. The velocity u_0 with which the jet strikes the vane is slightly less than u_1 because of the deceleration due to gravity. This effect may be calculated from the expression.

$$u_0^2 = u_1^2 - 2gs$$

Inserting the value s = 0.09 m leads to the result

$$u_o = \sqrt{u_1^2 - 1.7658} \text{ m/s}$$

Values of quantity collected, measured time, and jockey weight W are presented in Table 5.1 and 5.2, together with the ensuing calculations.

Mass flow rate $\dot{m}$ (20/ 36) = 0.555 kg/ s

Volumetric flow rate Q 5.55 * 10^{-4} m^3/ s

Velocity at nozzle exit u_1 = Q/A 5.555*10^{-4}/ 7.85*10^{-5} = 7.077 m/s

Velocity at impact with vane u_0 $\sqrt{u_1^2 - 1.7658}$ = 6.9511 m/s

Momentum flow in jet at impact J = $\dot{m}u_0$ = 0.555 * 6.9511 = 3.8579 N

Force on vane F 7.7822 N

Qty (kg)	**t (s)**	$\dot{m}$ **(kg/s)**	**W (N)**	**u_1 (m/s)**	**u_0 (m/s)**	**J (N)**	**F (N)**

Table 5.1 Results for flat plate

Qty (kg)	**t (s)**	$\dot{m}$ **(kg/s)**	**W (N)**	**u_1 (m/s)**	**u_0 (m/s)**	**J (N)**	**F (N)**

Plot Fore on Vane (N) as a function of Rate of momentum flow in jet J (N) for both types of vane and find out the slope.

Discussion of Results:

It is clear from the above graphs that the force produced on each of the vanes is proportional to the momentum flow in the jet as it strikes the vane.

For the flat plate, the slope of the graph is , as compared with the ideal value of 1.00. The discrepancy is possibly due to systematic errors, such as a small error in the measured value of nozzle diameter. A further possibility lies in the behavior of the jet after striking the vane. It forms a radial sheet which impinges on the inner wall of the surrounding cylinder, and then divides, some of the water flowing down the cylinder wall and the rest flowing upwards. Although visibility is impaired by the spray which is generated, it does seem that some water falls on to the top side of the vane. This would have the effect of producing a small momentum force in the downwards direction, so reducing the net upwards force on the vane.

For the hemispherical cup, the slope of the graph is , so that

$$F = \quad * J$$

The maximum possible force is shown by equation **(5.8)** to be 2J, which occurs when the jet is deflected through 180° without energy loss. The ratio between measured force and the theoretical maximum may be regarded as an efficiency of the vane and will always be less than 1.00 because losses will always occur in practice.

6. FREE & FORCED VORTEX

Introduction:

Bernoulli's equation states that, in the steady motion of an incompressible and inviscid fluid, the total head H is constant along a streamline. However, it gives no indication of how H varies from one streamline to another. In some flows, it may safely be assumed that the total head is constant across the streamlines as well as along them. For example, the orifice flow described in chapter 6 is satisfactorily treated on the basis of uniform total head everywhere in the motion. Consider, on the other hand, the Poiseuille flow described in Chapter 3. Although the piezometric head is constant over any chosen cross section, the velocity rises from zero at the wall to a maximum at the centre. Consequently, the total head H also changes from streamline to streamline across the section.

The variation of H across the streamlines of a fluid in steady motion is closely related to rotation of the flow. It transpires that, when H is constant from streamline to streamline, there is no rotation in the fluid. Such a flow is referred to as **irrotational**. Conversely, a fluid motion in which H varies across the streamline does possess rotation, and is referred to as **rotational**. It is easy to see that rotation exists in Poisueille flow. As fluid flows in parallel motion along two adjacent streamlines, the one nearer to the centre moves faster, so a small element of fluid rotates in the relative motion. In the uniform jet from an orifice, however, there is no such relative motion, which is therefore irrotational.

Because of the importance of these concepts in many engineering applications of fluid mechanics, it is useful to examine the particular case of flow which circulates about a fixed centre, often called vortex flow. As indicated in Fig. 7.1, the streamline pattern consists simply of a set of concentric circles. The velocity u may vary in any chosen manner with radius r, so we may write in general:

$$u = f(r) \qquad \textbf{(7.1)}$$

There are two cases of particular interest. In the so called **Forced Vortex.**

$$u = \omega r \qquad \textbf{(7.2)}$$

the whole body of fluid rotates as if solid, at angular speed ω. In the so called **free vortex**,

$$u = \frac{C}{r} \qquad \textbf{(7.3)}$$

in which C is a constant. A unique feature of this motion is that the total head H is independent of radius r, and the flow is therefore irrotational. Despite appearance to the contrary, elements of fluid do not rotate as they move around the centre of the motion.

Description of Apparatus:

The designed set up consists of transparent cylinder for free and forced vortex. The impeller in the case fitted with bearing which may be rotated about its axis with the help of the forced created by water itself. Cylinder for the free vortex has a hole in the centre of its base.

For generating forced vortex the cylinder with a hole (orifixce) in the bottom is rotated for several minutes until the whole body of water has picked up a uniform rotation from the walls of the cylinder. To create a free vortex, a small through flow is required. As the water passes into the cylinder, angular momentum is imparted to it, and it then flows in spiral motion towards the centre. After a while, a steady vortex is produced in which the surface level drops very steeply at the centre. The air core which is formed passes right down to the base of the cylinder and out through the hole at the centre. Apart from internal loss and energy transfer between the rotating cylinder and the fluid, the total head is constant throughout the motion, so the flow approximates that of a free vortex.

Experimental Procedure:

Forced Vortex

To create a forced vortex, the cylinder with hole in the bottom is placed over the stand and it is filled with water to approximately ¼ depth. The water gradually picks up the rotation, and after several minutes, becomes steady. To ensure that the steady condition has truly been reached, it is recommended that the central depth be taken at, say, one half minute intervals, until a repeated value is obtained over several successive observations.

Free Vortex

To form the free vortex, the cylinder with hole is placed over the stand. Water is then supplied from a suitable constant head source to it. Angular momentum is then imparted to the water and it then spirals in towards the centre, flowing out of the cylinder through the hole in the base. The cylinder is then set in fairly slow rotation at say 30 rev/ min, and the water supply is adjusted to give a steady depth of water in the cyl

Free & Forced Vortex Flows

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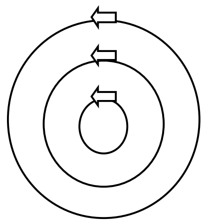
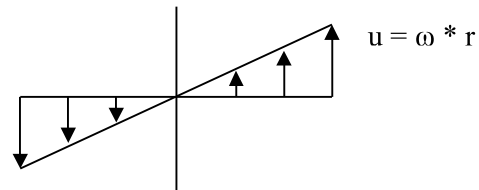
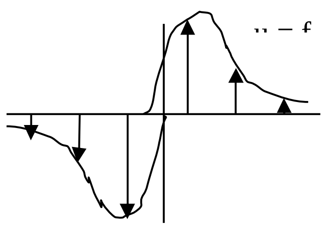
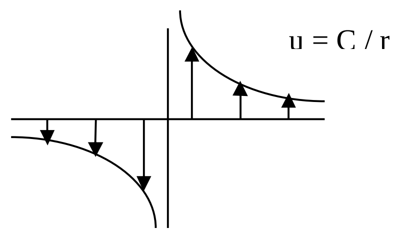


Streamline Pattern



Forced Vortex





General Velocity Distribution ***Free Vortex***

Illustration of Vortex Flows

The velocity **u** may vary in any chosen manner with radius **r**, so in general:

$$u = f(r)$$

There are two cases of particular interest. In the so-called **Forced Vortex,**

$$u = \omega * r$$

The whole body of fluid rotates as if solid, at angular speed ω. In the so called **Free Vortex,**

$$u = C / r$$

in which C is a constant. A unique feature of this motion is that the total head H is independent of radius r, and the flow is therefore irrotational. Despite appearance to the contrary, elements of fluid do not rotate as they move around the center of the motion.

DESCRIPTION OF APPARATUS:

The apparatus comprises a clear-transparent acrylic cylinder on a plinth designed to produce free and forced vortices. The free vortex is generated by water discharging through an interchangeable orifice in the base of the cylinder and the resulting profile is measured using a combined caliper and depth scale. The forced vortex is induced by a paddle in the base of the cylinder, which is rotated by jets of water. As the water passes tangentially into the cylinder, angular momentum is imparted to it, and it then flows in spiral motion towards the center. After a while, a steady vortex is produced in which the surface level drops very steeply at the center. The air core which is formed passes right down to the base of the cylinder and out through the hole at the center. Apart from internal loss, the total head is constant throughout the motion, so the flow approximates that of a free vortex. The profile of the vortex is determined using a series of depth gauges.

Operation:

AIM:

1. To understand the difference between free and forced vortices.
2. To determine surface profile of a forced vortex.
3. To determine the surface profile a free vortex.

EXPERIMENTAL PROCEDURE:

- **FORCED VORTEX:**

  To create a forced vortex, the hole in the base of the cylinder is closed by insertion of the plug provided for the purpose. The paddle in the base of the cylinder is rotated with help of tangential jet of water. As the water passes tangentially into the cylinder, angular momentum is imparted to it, and it then flows in spiral motion towards the center. After a while, a steady vortex is produced in which the surface level drops very steeply at the center. The air core which is formed passes right down to the base of the cylinder and out through the hole at the center.

FREE VORTEX:

To create a free vortex, the orifice in the base of the cylinder is attached and flow of water is started to flow. The free vortex is generated by water discharging through an interchangeable orifice in the base of the cylinder and the resulting profile is measured using a combined caliper and depth scale.

OBSERVATION:

Tank Diameter : 300 mm

Height to overflow point :

Total height of cylinder : 400 mm

Orifice Diameters :

Inlet Tube Dia. :

OBSERVATIONAL TABLE:

FREE VORTEX:

SR. NO.	H	R1	R2	R

FORCED VORTEX:

SR. NO.	H	R1	R2	R

CALCULATIONS:

RESULTS:

CONCLUSION:

DESCRIPTION OF APPARATUS:

The apparatus comprises a clear-transparent acrylic cylinder on a plinth designed to produce free and forced vortices. The free vortex is generated by water discharging through an interchangeable orifice in the base of the cylinder and the resulting profile is measured using a combined caliper and depth scale. The forced vortex is induced by a paddle in the base of the cylinder, which is rotated by jets of water. As the water passes tangentially into the cylinder, angular momentum is imparted to it, and it then flows in spiral motion towards the center. After a while, a steady vortex is produced in which the surface level drops very steeply at the center. The air core which is formed passes right down to the base of the cylinder and out through the hole at the center. Apart from internal loss, the total head is constant throughout the motion, so the flow approximates that of a free vortex. The profile of the vortex is determined using a series of depth gauges.

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- **FREE VORTEX:**
  To create a free vortex, the orifice in the base of the cylinder is attached and flow of water is started to flow. The free vortex is generated by water discharging through an interchangeable orifice in the base of the cylinder and the resulting profile is measured using a combined caliper and depth scale.

OBSERVATION:

Tank Diameter : 300 mm
Total height of cylinder : 350 mm
Orifice Diameters : 6, 10,12 mm

FREE VORTEX ORIFICE DIA.:	***FORCED VORTEX***

No.	**R**	**H**	**H_0-H**		**No.**	**R**	**H**	**H_0-H**
1		H_0=			1		H_0=	
2					2			
3					3			
4					4			
5					5			
6					6			
7					7			
8					8			
9					9			
10					10			
11					11			
12					12			
13					13			
14					14			
15					15			

GRAPH:

1. R VS (H_0 – H) for free vortex of different orifice
2. R VS (H_0 – H) for forced vortex of fixed rotation
3. **CONCLUSION:**

The results show that both the surface and the total head profiles in the forced vortex agree extremely well with the prediction of theory. The whole body of water rotates as if solid. In case of the free vortex, however, the results do not conform to the theory. Far from producing an ideal free (where the velocity decreases from the center in proportion to 1/ r.), the experiments create vortex in which the circulating velocity decreases as $r^{-0.385}$. in such a vortex, the total head H is not constant throughout, but falls towards the center; the variation of H with radius r has been derived from the experimental measurement of the surface profile.

9. BERNOULLI'S THEOREM

Introduction:

Energy of a Liquid in Motion:

The energy, in general, may be defined as the capacity to do work. Though the energy exists in many forms, yet the following are important from the subject point of view:

1. Potential Energy
2. Kinetic Energy, and
3. Pressure Energy.

Potential Energy of a Liquid in Motion:

It is the energy possessed by a liquid particle, by virtue of its position. If a liquid particle is Z meters above the horizontal datum (arbitrarily chosen), the potential energy of the particle will be Z meter-kilogram (briefly written as mkg) per kg of liquid. Potential head of the liquid, at that point, will be Z meters of the liquid.

Kinetic Energy of a Liquid Particle in Motion:

It is the energy, possessed by a liquid particle, by virtue of its motion or velocity. If a liquid particle is flowing with a mean velocity of v meter per second, then the kinetic energy of the particle will be $v^2/ 2g$ mkg per kg of liquid. Velocity head of the liquid, at that velocity, will be $v^2/ 2g$meter of the liquid.

Pressure Energy of a Liquid Particle in Motion:

It is the energy, possessed by a liquid particle, by virtue of its existing pressure. If a liquid particle is under a pressure of p kg / m^2, then the pressure energy of the particle will be p/w mkg per kg of liquid, where w is the specific weight of the liquid. Pressure head of the liquid under that pressure will be p/w meter of the liquid.

Total Energy of a Liquid Particle in Motion:

The total energy of a liquid particle, in motion, is the sum of its potential energy, kinetic energy and pressure energy. Mathematically,

Total Energy,

$$E = Z + \frac{v^2}{2g} + \frac{p}{w} \text{ mkg/ kg of liquid} \qquad \textbf{(8.1)}$$

Bernoulli's Equation:

It states, "For a perfect incompressible liquid, flowing in a continuous stream, the total energy of a particle remains the same; while the particle moves from one point to another." This statement is based on the assumption that there is no losses due to friction in pipe. Mathematically,

$$E = Z + \frac{v^2}{2g} + \frac{p}{w} = \text{constant}$$

(8.2)

Consider a perfect incompressible liquid, flowing through a nonuniform pipe as shown in Figure 8.1

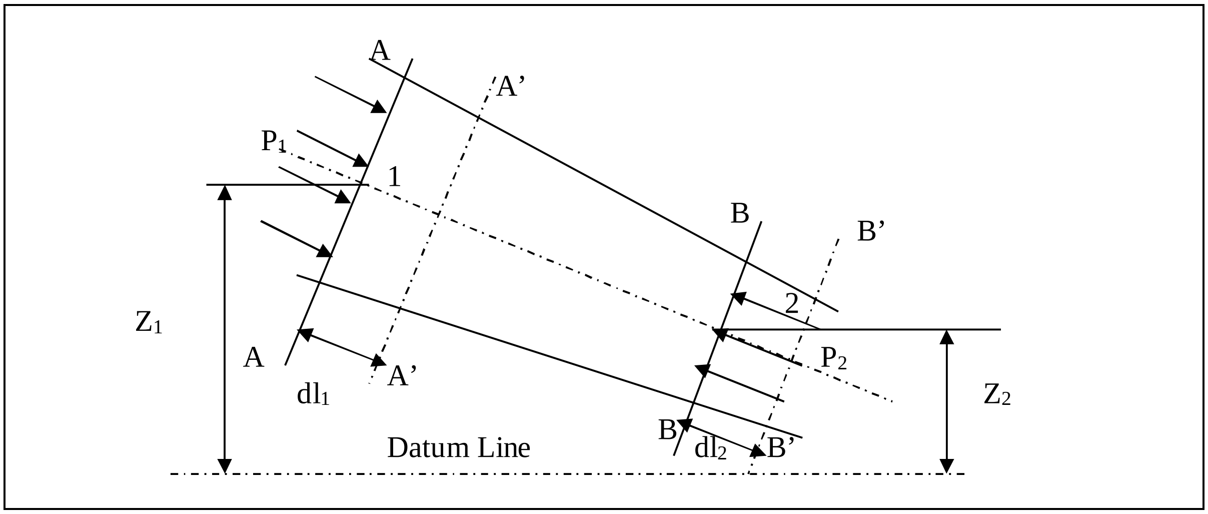


Figure 8.1 Bernoulli's Equation

Let us consider two cross sections AA and BB of the pipe. Now let us assume that the pipe is running full and there is a continuity of flow between the two sections.

Let

Z_1 = Height of AA above the datum
p_1 = Pressure at AA
v_1 = Velocity of liquid at AA
a_1 = Area of pipe at AA

Z_2, p_2, v_2, a_2 = corresponding values at BB

Let the liquid between the two sections AA and BB move to A'A' and B'B' through very small lengths dl_1 and dl_2 as shown in figure 8.1. This movement of the liquid between AA and A'A' to Bb and B'B', the remaining liquid between A'A' and BB being unaffected.

Let W be the weight of the liquid between AA and A'A'. Since the flow is continuous, therefore

$$W = wa_1dl_1 = wa_2dl_2$$

or

$$a_1dl_1 = W/w \tag{8.3}$$

Similarly

$$a_2dl_2 = W/w$$

Therefore,

$$a_1dl_1 = a_2dl_2 \tag{8.4}$$

Work done by pressure at AA, in moving the liquid to A'A'

$$= \text{Force} * \text{Distance} = p_1a_1dl_1$$

Similarly, work done by pressure at BB in moving the liquid to B'B'

$$= -\,p_2a_2dl_2$$

(minus sign is taken as the direction of p_2 is opposite to that of p_1).

Total work done by the pressure

$$= p_1a_1dl_1 - p_2a_2dl_2$$

$$= p_1a_1dl_1 - p_2a_1dl_1$$

(from eqn. 8.4)

$$= a_1dl_1\,(p_1 - p_2)$$

$$= W/w\,(p_1 - p_2)$$

(from eqn. 8.3)

Loss of potential energy

$$= W\,(Z_1 - Z_2)$$

and again in kinetic energy

$$= W\left(\frac{v_2^2}{2g} - \frac{v_1^2}{2g}\right) = \frac{W}{2g}\left(v_2^2 - v_1^2\right)$$

we know that

Loss of potential energy + work done by pressure = Gain in kinetic energy

$$W\,(Z_1 - Z_2) + W/w\,(p_1 - p_2) = \frac{W}{2g}\left(v_2^2 - v_1^2\right)$$

$$(Z_1 - Z_2) + \frac{p_1}{w} - \frac{p_2}{w} = \frac{v_2^2}{2g} - \frac{v_1^2}{2g}$$

or

$$Z_1 + \frac{v_1^2}{2g} + \frac{p_1}{w} = Z_2 + \frac{v_2^2}{2g} + \frac{p_2}{w}$$

Which proves the **Bernoulli's equation.**

Description of Apparatus:

The apparatus is made from transparent acrylic and has both the convergent and divergent sections. Water is supplied from the constant head tank attached to the test section. Constant level is maintained in the supply tank using adjustable overflow system. Piezometric tubes are attached at equal distance on the test section. Ware discharges to the discharge tank attached at the far end of the test section and from there it goes to the weighing tank through valve. The entire setup is mounted on a stand.

Experimental Procedure:

Set the setup over the tray and connect the overflow line provided at the bottom of the supply tank to the liquid level adjuster. Another end of the liquid level adjuster discharges the water in to the sump tank. Flexible supply hose is connected to the supply line and also to the drain line in the discharge tank. The bench supply valve is then opened and adjusted until there is a steady level of liquid in the supply tank. Keep drain valve of the discharge tank fully open. After we get steady height of liquid in the supply tank adjust the drain valve on the discharge tank so as to get steady level there also.

Collect the predetermined quantity of water in the weighing tank and measure the time required for the same. Also record the height of liquid in each of the piezometric tube.

Results and Calculations:

Length of the test section	800 mm	0.8 m
Length of the divergent section	300 mm	0.3 m
Length of the convergent section	500 mm	0.5 m
Cross sectional area at the beginning of the divergent section		$7.5 * 10^{-4}\ m^2$
Cross sectional area at the throat section		$3.75 * 10^{-4}\ m^2$
Cross sectional area at the end of the convergent section		$7.5 * 10^{-4}\ m^2$
Distance between successive piezometric tube		0.1 m

Sr. No.	Qty (kg)	t (s)	Q (kg/ s)	Height / Pressure head of liquid in the piezometric tube (m of water column)						
				p_1/w	p_2/w	p_3/w	p_4/w	p_5/w	p_6/w	p_7/w

Sr. No.	Qty (kg)	t (s)	Q (kg/ s)	Velocity head of liquid at the piezometric tube (m of water column)						
				$v_1^2/ 2g$	$v_2^2/ 2g$	$v_3^2/ 2g$	$v_4^2/ 2g$	$v_5^2/ 2g$	$v_6^2/ 2g$	$v_7^2/ 2g$

Check for all the points that sum of velocity head and pressure head is almost constant. Here potential energy is not being considered as the test section lies in one horizontal plane only.

Sr. No.	Qty (kg)	t (s)	Q (kg/ s)	Total head of liquid at the piezometric tube (m of water column) ($p/w + v^2/ 2g$)						

Limitations of Bernoulli’s Equation:

The Bernoulli’s theorem or Bernoulli’s equation has been derived on certain assumptions, which are rarely possible. Thus the Bernoulli’s theorem has the following limitations.

1. The Bernoulli’s equation has been derived under the assumption that the velocity of every liquid particle, across any cross section of a pipe, is uniform. But in actual practice, it is not so. The velocity of liquid particle in the centre of a pipe towards the walls of a pipe is maximum, and gradually decreases while using the Bernoulli’s equation, only the mean velocity of the liquid should be taken into the account.

2. The Bernoulli’s equation has been derived under the assumption, that no external force, except the gravity force is acting on the liquid. But in actual practice, it is not so. There are always some external forces (such as pipe friction etc.) acting on the liquid, which effect the flow of the liquid. Thus while using the Bernoulli’s equation, all such external forces should be neglected. But if some energy is supplied to, or extracted from the flow, the same should be taken into account.

3. The Bernoulli’s equation has been derived, under the assumption, that there is no loss of energy of the liquid particle while flowing. But in actual practice, it is rarely so, in a turbulent flow some kinetic energy is converted into heat energy; and in a viscous flow some energy is converted due to shear forces. Thus while using Bernoulli’s equation all such losses should be neglected.

4. If the liquid is flowing in a curved path, the energy due to centrifugal force should also be taken into account.

10. METACENTRIC HEIGHT APPARATUS

INTRODUCTION:

For the investigation of stability of floating body, it is necessary to determine the position of its metacentric with respect to its center of gravity. Consider a floating ship model, the weight of the ship acts through its center of gravity and is balanced by an equal and opposite buoyant force acting upwards through the center of buoyancy. i.e. the center of gravity of liquid displaced by the floating body.

A small angular displacement shifts the center of buoyancy and the intersection of the line of action of the buoyant force passing through the new center of buoyancy and the extended line would give the metacentre.

The distance between center of gravity and metacentre is known as metacentric height.

There are three conditions of equilibrium of a floating body,

Stable Equilibrium -	Metacenter lies above the center of gravity
Unstable Equilibrium -	Metacenter lies below the center of gravity
Neutral Equilibrium -	Metacenter coincides with center of gravity

The metacentric height GN is given by,

$$\mathbf{GM = (m * X)/(W * \tan\theta)}$$

Where,

W = weight of the floating body
m = movable weight
X = distance through which the movable load is shifted
θ = Angle of heel

APPARATUS:

The apparatus consist of a tank and is provided with a drain cock and self graduated level indicator. The floating body is made from sheet metal .It is provided with movable weights, protractor to measure the angle of heel and pointer. Weights are also provided to increase the weight of floating body by known amount.

AIM:

To determine the metacentric height of a given floating body.

PROCEDURE:

- Fill the acrylic tank to about 2/3 levels and note the level of water from the base of tank.
- Place the floating body in the tank and note the increase the level of water.
- Calculate the Weight of water displaced, which is equal to the weight of floating body.
- Apply momentum to the floating body by moving one of the adjustable weights (m) through a known distance.
- Note down the angle of heel corresponding to this shift of weight with the help of protractor and pointer.
- Take about 4-5 such readings by changing the position of the adjustable weight and find out center of gravity in each case.

OBSERVATION:

Initial level of water in the tank = cm
Final level of water in the tank = cm
Weight of the ship model = N
C/ S Area of the tank = cm^2

Sr. No.	**Movable Weight (m) N**	Distance moved **(X) m**	**Angle of Tilt (θ)**	**tan θ**	**Metacentric Height GM M**
1					
2					
3					
4					
5					

Result:

Conclusion:

SAMPLE CALCULATION FOR METACENTRIC HEIGHT APPARATUS:

OBSERVATION:

Initial level of water in the tank = 24 cm
Final level of water in the tank = 25 cm
Weight of the acrylic pontoon = 3.0 kgs
C/ S Area of the tank = 3000 cm^2

Sr. No.	**Movable Weight (m) N = m.g**	Distance moved **(X) m**	**Angle of Tilt (θ)**	**tan θ**	**Metacentric Height GM M**
1					
2					

The metacentric height GM is given by,

Reading: 2

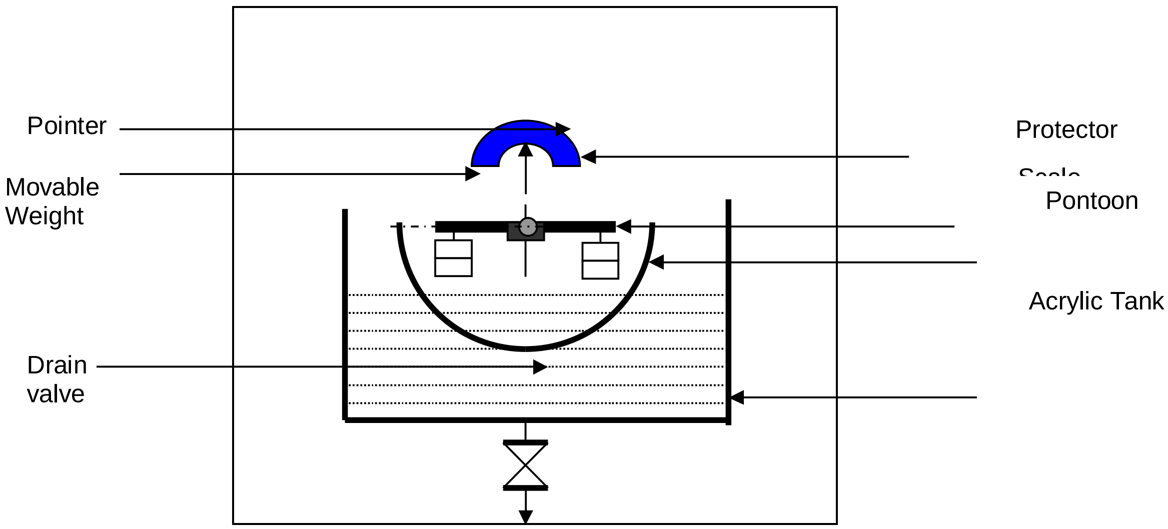
GM = (m * X)/ (W * tan θ)

=

Where,

W = weight of the floating body
m = movable weight
X = distance through which the movable load is shifted
θ = Angle of heel

STABILITY OF FLOATING BODIES
METACENTRIC HEIGHT APPARATUS



METACENTRIC HEIGHT APPARATUS

INTRODUCTION:

For the investigation of stability of floating body, it is necessary to determine the position of its Metacentric with respect to its center of gravity. Consider a floating ship model, the weight of the ship acts through its center of gravity and is balanced by an equal and opposite buoyant force acting upwards through the center of buoyancy. I.e. the center of gravity of liquid displaced by the floating body.

A small angular displacement shifts the center of buoyancy and the intersection of the line of action of the buoyant force passing through the new center of buoyancy and the extended line would give the metacentre. The distance between center of gravity and metacentre is known as Metacentric height.

There are three conditions of equilibrium of a floating body,

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The Metacentric height GN is given by,

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Where,

W = weight of the floating body

m = movable weight

X = distance through which the movable load is shifted

θ = Angle of heel

EXPERIMENTAL MANUAL

APPARATUS:

The apparatus consist of acrylic tank and is provided with a drain cock and self graduated level indicator. The floating body is also made from Acrylic. It is provided with movable weights, protractor to measure the angle of heel and pointer. Weights are also provided to increase the weight of floating body by known amount.

OPERATIONAL MANUAL

AIM:

To determine the Metacentric height of a given floating body.

PROCEDURE:

⇒ Fill the Acrylic tank to about 75 % levels and note the level of water from the base of tank.

⇒ Place the floating body in the tank and note the increase the level of water.

⇒ Calculate the Weight of water displaced, which is equal to the weight of floating body.

⇒ Apply momentum to the floating body by moving one of the adjustable weights (m) through a known distance.

⇒ Note down the angle of heel corresponding to this shift of weight with the help of protractor and pointer.

⇒ Take about 4-5 such readings by changing the position of the adjustable weight and find out center of gravity in each case.

OBSERVATION:

Initial level of water in the tank = m

Final level of water in the tank = m

Weight of the ship model = Kg

C / S Area of the tank = m^3

Sr. No.	**Movable Weight (m) Kg**	Distance moved **(X) m**	**Angle of Tilt (θ)**	**tan θ**	**Metacentric Height GN M**
1					
2					
3					
4					
5					

Calculation:

W = C/S Area of the tank * Density of water

=

=

GN = (m * X) / (W * tanθ)

=

=

=

=

Result:

Conclusion:

<u>SAMPLE CALCULATION</u>

OBSERVATION:

- Initial level of water in the tank = 304 mm.
- Final level of water in the tank = 313 mm.
- Weight of the ship model = 2.1 Kg
- C / S Area of the tank = 600 *403 mm^3

Sr. No.	**Movable Weight (m) Kg**	Distance moved **(X) m**	**Angle of Tilt (θ)**	**tan θ**	**Metacentric Height GN m**
1					
2					
3					
4					

Sample calculation: For reading 1:

W = C/S Area of the tank * Density of water

GN = (m * X) / (W * tan θ)

=