Question Bank (Mid Examination)
CVPDE (3130005)

1. $Find\ the\ moduli\ and\ argument\ \&\ principal\ argument\ \ of\ z = \frac{1-i}{1+i}.$

2. $Find\ the\ real\ and\ imaginary\ parts\ and\ principal\ argument\ of\ f(z) = \frac{2+6\sqrt{3}\,i}{5+\sqrt{3}\,i}.$

3. $State\ De - Movire's\ formula\ and\ hence\ evaluate\ \left(1 + i\sqrt{3}\right)^{100} + \left(1 - i\sqrt{3}\right)^{100}.$

4. $If\ x_r = cos\frac{\pi}{2^r} + isin\frac{\pi}{2^r}, prove\ that\ x_1.x_2.x_3 \ldots\ldots\infty = -1.$

5. $Find\ real\ and\ imaginary\ part\ of\ (-1-i)^7 + (-1+i)^7.$

6. $Find\ the\ distinct\ values\ of\ z = (1+i)^{\frac{1}{3}}.$

7. $Prove\ that\ the\ n^{th}\ roots\ of\ unity\ are\ in\ geometric\ progression\ with\ common\ ratio\ \cos\frac{2\pi}{n} + i\sin\frac{2\pi}{n}. Also\ show\ that\ their\ sum\ is\ zero.$

8. $Find\ all\ the\ values\ \ of\ \left(\frac{1}{2} + \frac{\sqrt{3}}{2}\,i\right)^{3/4}. Also\ find\ their\ product.$

9. $Let\ z = x + iy.\ Showing\ details, find, in\ terms\ of\ x\ and\ y: (i)\ Im\left(\frac{1}{z}\right)(ii) Im\left(\frac{1}{\bar{z}^2}\right)$

10. $Find\ and\ graph\ all\ roots\ in\ the\ complex\ plane.\ \ (i)\ \sqrt[5]{-1}\ \ (ii)\ \sqrt[4]{i}$

11. *Find principal value of following* $(i)\log(1+i)\ (ii)\log(-i)\ \ (iii) log\ 1\ (iv)(3)^{4-i}\ \&\ (2i)^{2i}$

12. *Find real and imaginary parts of following*
$(i)\sin z\ \ (ii)\sinh z\ \ (iii)(1+i)^i\ (iv\ )(-i)^i$

13. Prove that $u = x^2 - y^2 + e^x\cosh y$ is harmonic and find its analytic function and harmonic conjugate.

14. $State\ \ Cauchy - Riemann\ Equations. Show\ that$
$(I) f(z) = sin\,z\ is\ everywhere\ analytic\ (II)\ f(z) = xy + iy\ is\ nowhere\ analytic$

15. $Separate\ real\ and\ imaginary\ parts\ of\ f(z) = e^z. Also\ prove\ that\ it\ is\ analytic\ everywhere.$

Power Series & Radius of convergence

16. $Expand\ f(z) = \frac{1}{(z+2)(z+4)}\ \ in\ the\ region\ (i)|z| < 2\ (ii) 2 < |z| < 4\ (iii)|z| > 4$

17. $Expand\ f(z) = \frac{1}{z(1-z)}\ \ in\ the\ region\ (i)|z+1| < 1\ (ii) 1 < |z+1| < 2\ (iii)\ |z+1| > 2$

18. $Expand\ f(z) = \frac{1}{z^2-4z+3}\ \ in\ the\ region\ (i) 1 < |z| < 2.5\ (ii) 1 < |z-1| < 2\ (iii)\ |z| > 3$

19. $Find\ radius\ \&\ region\ of\ convergence\ of\ follwing$
$\sum_{n=1}^{n=\infty}(3+4i)^n z^n,\ \sum_{n=1}^{n=\infty}\left(\frac{(n!)^2}{(2n)!}\right)z^n,\ \sum_{n=1}^{n=\infty}\left(\frac{6n+1}{2n+5}\right)(z-2i)^n, \sum_{n=1}^{n=\infty}\frac{n!}{n^2}z^n,$
$\sum_{n=0}^{n=\infty} n(n-1)2^n z^{3n},\ \sum_{n=0}^{n=\infty}\frac{z^n}{n!}.\ \sum_{n=0}^{n=\infty}\frac{(-1)^n}{(2n+)!}z^{2n+1}.$

Conformal mapping & Bilinear(Mobius) Transform

20. Find fixed point of following $w = \frac{1+z}{1-z}, w = \frac{1}{z+2i}$.

21. $Find\ the\ bilinear\ transformation\ that\ maps\ the\ points\ z_1 = -1, z_2 = 0, z_3 = 1$ $onto\ the\ points\ w_1 = -i, w_2 = 1, w_3 = i\ respectively. Also\ find\ w\ for\ z = \infty.$

22. $Find\ the\ bilinear\ transformation\ which\ transforms\ z_1 = \infty, z_2 = i, z_3 = 0\ into\ w_1 = 0,\ w_2 = i, w_3 = \infty\ .$

23. $Find\ the\ image\ of\ |z + i| = 1\ under\ the\ mapping\ w = \frac{1}{z}.$

24. $Determine\ the\ region\ in\ w - plane\ into\ which\ the\ region\ bounded\ by\ x = 0, y = 0, x = 1, y = 2\ in\ the\ z - plane\ is\ mapped\ under\ transformation$ $w = (1 + i)z + 2 - i$

25. $Find\ the\ image\ of\ semi - infinite\ strip\ x > 0, 0 < y < 2, where\ w = iz + 1.$ $Sketch\ the\ strip\ and\ its\ image.$

Partial Differential Equations (PDE)

26. *Find partial differential equation of* $i.\ z = ax^2 + by^2\quad ii.\ z = (x + a)(y + b)\quad iii.\ z = (x - a)^2 + (x - b)^2\quad iv\ z = f(x^2 + y^2)$

27. Solve $(y + z)p + (z + x)q = x + y.$

28. Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz.$

29. Solve partial differential equation $px + qy = pq$

30. Solve $p^2 + q^2 = npq$