

Question Banks

CVPDE (3140610)

1. Find the moduli and argument & principal argument of $z = \frac{1-i}{1+i}$.
2. Find the real and imaginary parts and principal argument of $f(z) = \frac{2+6\sqrt{3}i}{5+\sqrt{3}i}$.
3. State De – Moivre’s formula and hence evaluate $(1 + i\sqrt{3})^{100} + (1 - i\sqrt{3})^{100}$.
4. If $x_r = \cos \frac{\pi}{2^r} + i \sin \frac{\pi}{2^r}$, prove that $x_1 \cdot x_2 \cdot x_3 \dots \dots \infty = -1$.
5. Find real and imaginary part of $(-1 - i)^7 + (-1 + i)^7$.
6. Find the distinct values of $z = (1 + i)^{\frac{1}{3}}$.
7. Prove that the n^{th} roots of unity are in geometric progression with common ratio $\cos \frac{2\pi}{n} + i \sin \frac{2\pi}{n}$. Also show that their sum is zero.
8. Find all the values of $\left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{3/4}$. Also find their product.
9. Let $z = x + iy$. Showing details, find, in terms of x and y : (i) $\text{Im}\left(\frac{1}{z}\right)$ (ii) $\text{Im}\left(\frac{1}{z^2}\right)$
10. Find and graph all roots in the complex plane. (i) $\sqrt[5]{-1}$ (ii) $\sqrt[4]{i}$
11. Find principal value of following (i) $\log(1 + i)$ (ii) $\log(-i)$ (iii) $\log 1$
12. Find real and imaginary parts of following
(i) $\sin z$ (ii) $\sinh z$ (iii) $(1 + i)^i$ (iv) $(-i)^i$
13. Prove that $u = x^2 - y^2 + e^x \cosh y$ is harmonic and find its analytic function and harmonic conjugate.
14. State Cauchy – Riemann Equations. Show that
(I) $f(z) = \sin z$ is everywhere analytic (II) $f(z) = xy + iy$ is nowhere analytic
15. Separate real and imaginary parts of $f(z) = e^z$. Also prove that it is analytic everywhere.

Power Series & Radius of convergence

16. Expand $f(z) = \frac{1}{(z+2)(z+4)}$ in the region (i) $|z| < 2$ (ii) $2 < |z| < 4$ (iii) $|z| > 4$
17. Expand $f(z) = \frac{1}{z(1-z)}$ in the region (i) $|z + 1| < 1$ (ii) $1 < |z + 1| < 2$ (iii) $|z + 1| > 2$
18. Find radius & region of convergence of following
 $\sum_{n=1}^{\infty} (3 + 4i)^n z^n$, $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n)!} z^n$, $\sum_{n=1}^{\infty} \frac{(6n+1)}{(2n+5)} (z - 2i)^n$, $\sum_{n=1}^{\infty} \frac{n!}{n^2} z^n$,
 $\sum_{n=0}^{\infty} \frac{z^n}{n!}$.

Conformal mapping & Bilinear(Mobius) Transform

19. Find fixed point of following $w = \frac{1+z}{1-z}$, $w = \frac{1}{z+2i}$.

20. Find the bilinear transformation that maps the points $z_1 = -1, z_2 = 0, z_3 = 1$ onto the points $w_1 = -i, w_2 = 1, w_3 = i$ respectively. Also find w for $z = \infty$.
21. Find the bilinear transformation which transforms $z_1 = \infty, z_2 = i, z_3 = 0$ into $w_1 = 0, w_2 = i, w_3 = \infty$.
22. Find the image of $|z + i| = 1$ under the mapping $w = \frac{1}{z}$.
23. Determine the region in w - plane into which the region bounded by $x = 0, y = 0, x = 1, y = 2$ in the z - plane is mapped under transformation $w = (1 + i)z + 2 - i$
24. Find the image of semi - infinite strip $x > 0, 0 < y < 2$, where $w = iz + 1$. Sketch the strip and its image.

Partial Differential Equations (PDE)

25. Find partial differential equation of i. $z = ax^2 + by^2$ ii. $z = (x + a)(y + b)$ iii. $z = (x - a)^2 + (x - b)^2$ iv. $z = f(x^2 + y^2)$ v. $f(xy + z^2, x + y + z) = 0$ vi. $\phi(x + y + z, x^2 + y^2 + z^2) = 0$.
26. Solve $(y + z)p + (z + x)q = x + y$ & $\frac{y-z}{yz}p + \frac{z-x}{zx}q = \frac{x-y}{xy}$.
27. Solve $(x^2 - y^2 - z^2)p + 2xyq = 2xz$, $x^2p + y^2q = z^2$, & $x(y - z)p + y(z - x)q = z(x - y)$
28. Solve partial differential equation $px + qy = pq$
29. Solve $p^2 + q^2 = npq$
30. Solve $px + qx = pq$.
31. Solve $z = px + qy + p^2 \log q$
32. Solve $p + q + pq = 0, p^2 + 3q = 0$
33. Solve $p - q = -y + x$ & $p^2 - x = q^2 - y$