Tutorial 01(a)

1. *Express the matrix A as sum of a Hermitian and skew-Hermitian matrix where*

$$A = \begin{bmatrix} 4+2i & 7 & 3-i \\ 0 & 3i & -2 \\ 5+3i & -7+i & 9+6i \end{bmatrix}$$

2. *Solve the following system for x, y and z*
   $x + y + 2z = 9, 2x + 4y - 3z = 1, 3x + 6y - 5z = 0$
3. *Consider the following system:*
   $x + 2y + z = 8, 2x + 2y + 2z = 13, 3x + 4y + vz = w$. *For what values of v and w do the system has (i) no solution (ii) unique solution (iii) infinite solution.*
4. *Solve the following equations by Gaussian elimination method:*
   $2x + y + 2z = 9,\ \ 2x + 4y - 3z = 1,\ \ 3x + 6y - 5z = 0$.
5. *Find the rank of the matrix* $A = \begin{bmatrix} 1 & 2 & -3 & 1 \\ -1 & 0 & 3 & 4 \\ 0 & 1 & 2 & -1 \\ 2 & 3 & 0 & -3 \end{bmatrix}$
6. *Determine the rank of the matrix A, if* $A = \begin{bmatrix} 3 & 4 & 5 & 6 & 7 \\ 4 & 5 & 6 & 7 & 8 \\ 5 & 6 & 7 & 8 & 9 \\ 10 & 11 & 12 & 13 & 14 \\ 15 & 16 & 17 & 18 & 19 \end{bmatrix}$.
7. *Test for consistency and solve*
   $4x - 2y + 6z = 8, x + y - 3z = -1, 15x - 3y + 9z = 21.$
8. *By using Gauss-Jordan Method, find the inverse of the matrix*
   $A = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$
9. *Find the inverse of the matrix A by Gauss-Jordan method* $A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$.

Tutorial 01(b)
Eigenvalues & Eigenvector
Formulas & Working rules

- *The characteristics equation of the matrix A of order 2 can be obtained from*

$$\boldsymbol{\lambda^2 - S_1\lambda + S_2 = 0}$$

Where S_1 *= sum of principal diagonal elements and*

$S_2 = Determinant\ of\ A$

- *The characteristics equation of the matrix A of order 3 can be obtained from*

$$\boldsymbol{\lambda^3 - S_1\lambda^2 + S_2\lambda - S_3 = 0}$$

Where $S_1 = sum\ of\ principal\ diagonal\ elements,$

$S_2 = sum\ of\ minors\ of\ principal\ diagonal\ elements\ and$

$S_3 = Determinant\ of\ A$

1. *Show that the characteristic equation of a* 2×2 *matrix* A *is given by* $\lambda^2 - S_1\lambda + |A| = 0$, *where* $S_1 = trace(A)$.

2. *If* λ *is an Eigen value of A then* $k\lambda$ *is also an Eigenvalue of* kA, *where* k *is a nonzero scalar, and* $\lambda + k$ *is an Eigen value of* $A + kI$.

3. *Find the Eigen values and Eigen vectors of the matrix* $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$.

4. *Verify Cayley-Hamilton theorem for* $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$, *and use it to find the simplified form of* $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I_3$.

5. *Find a matrix P that diagonalizes A, and determine* $P^{-1}AP$, *where* $A = \begin{bmatrix} 1 & 0 \\ 6 & -1 \end{bmatrix}$.

6. *Define: Algebraic multiplicity of an Eigen values. Determine algebraic and geometric multiplicity of each eigen value of the matrix.* $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$

7. $Find\ the\ eigen\ values\ \&\ bases\ for\ the\ eigenspace\ of\ A = \begin{bmatrix} 0 & 0 & -2 \\ 1 & 2 & 1 \\ 1 & 0 & 3 \end{bmatrix}$.

8. *Find a matrix P that diagonalizes* $A = \begin{bmatrix} 1 & 0 \\ -1 & 2 \end{bmatrix}$ *and hence find* A^{10}. *Also find the eigen values of* A^2.