COMPLEX VARIABLES AND PARTIAL DIFFERENTIAL EQUATIONS
(3140610)

TUTORIAL BOOK

Government Engineering College- Dahod

CERTIFICATE

This is to certify that Mr./Ms. ....

Enrolment No. .... of Second Year B.E., Semester ....

Branch ...., has satisfactorily completed his/her term work in course of Complex Variables & Partial differential Equations (3130005) for the academic year 20......- 20..... .

Date of Submission: ....

Signature of Faculty

Syllabus

$\boldsymbol{Complex\ Variables\ and\ Partial\ Differential\ Equations}$

$(\boldsymbol{Code}: -\mathbf{3140610})$

Complex Numbers and Analytic Functions: Weightage:-28%

- Polar Form of Complex Numbers,
- Powers and Roots,
- Complex Variable – Differentiation : Differentiation,
- Cauchy-Riemann equations, analytic functions, harmonic functions, finding harmonic conjugate
- Elementary analytic functions (exponential, trigonometric, logarithm) and their properties
- Conformal mappings,
- Mobius transformations and their properties

Complex Variable Integration & Power Series: Weightage:-20%

- Contour integrals, Cauchy-Goursat theorem (without proof), Cauchy Integral formula (without proof),
- Liouville's theorem and Maximum-Modulus theorem (without proof)
- Sequences, Series, Convergence Tests, Power Series, Functions Given by Power Series, Taylor and Maclaurin Series, Uniform Convergence

Residues & Laurent's series Weightage:-14%

- Laurent's series
- Zeros of analytic functions, singularities,
- Residues, Cauchy Residue theorem (without proof), Residue Integration Method, Residue Integration of Real Integrals

Partial differential equations: Weightage:-14%

- First order partial differential equations,
- Solutions of first order linear and nonlinear PDEs, Charpit's Method

Homogeneous and nonhomogeneous linear PDE: Weightage:-24%

- Solution to homogeneous and nonhomogeneous linear partial differential equations second and higher order by complementary function and particular integral method.
- Separation of variables method to simple problems in Cartesian coordinates, second-order linear equations and their classification,
- Initial and boundary conditions,
- Modeling and solution of the Heat,
- Wave and Laplace equations.

DEPARTMENT OF GENERAL

Action Plan For Students

- Minimum 75% attendance is compulsory for all the students for appearing in internal theory and practical exams
- Students must inform respective faculties if they remain absent continuously for particular reason that means if students remain absent for medical reason he/she needs to submit medical report.
- Every student must complete their assignment, tutorial, journal in given time. Students have to solve the examples from the previous question papers(which have been uploaded on college website) related to chapter and get it signed on same date. It will be not signed if the students failed to complete it in the given time limits.
- Grading system for tutorial, assignment, journal will be implemented for continuous evaluation of the students.
- Chapter wise exam will be taken and it will be compulsory for all students.
- Student must remain present with complete assignment, tutorial, journal and project on the scheduled date and time of internal submission. After scheduled time they will not be entertained at any cost.
- Queries related to subject will be solved during 10:30 am to 12:30 am every working Saturday. So students may remain present for queries

Index

Sr. No.	Tutorial No.	COs	Marks	Date	Signature
1.	Tutorial 01	CO 1			
2.	Tutorial 02	CO2			
3.	Tutorial 03	CO 2			
4.	Tutorial 04	CO 1			
5.	Tutorial 05	CO 2			
6.	Tutorial 06	CO 3			
7.	Tutorial 07	CO 3			
8	Tutorial 08	CO 4			
9	Tutorial 09	CO 4			

Tutorial 01
Complex numbers, polar form & roots

Formulas

- $Complex\ Number\ z = x + iy\quad where\ x = Re(z)\ and\ y = Im(z)$
- $i^2 = -1$
- $If\ Re(z) = 0\ then\ z = iy\ is\ called\ purely\ imaginary\ numbers,$
- $If\ Im(z) = 0\ then\ z = x\ is\ called\ purely\ real\ number$
- $\bar{z} = x - iy\ is\ called\ complex\ conjugate\ of\ z$
- $z\bar{z} = (x + iy)(x - iy) = x^2 + y^2$
- $To\ calculate\ principal\ value\ of\ argument$

  (i) $If\ x > 0, y > 0; argz = \tan^{-1}\frac{y}{x}$

  (ii) If x<0,y>0;arg z=π-$\tan^{-1}\left|\frac{y}{x}\right|$

  (iii) If x<0,y<0;argz =$\tan^{-1}\left|\frac{y}{x}\right| + \pi\ or - \pi + \tan^{-1}\left|\frac{y}{x}\right|$

  (iv) If x>0,y<0;arg z=$-\tan^{-1}\left|\frac{y}{x}\right|$
- De'moivre's Theorem: For any real number
  $(cos\theta + isin\theta)^n = cosn\theta + isinn\theta$
- Distinct complex roots of a complex number $z = r(\cos\theta + i\sin\theta)$ are

  $$[r(cos\theta + isin\theta)]^{\frac{1}{n}} = r^{\frac{1}{n}}\left(\cos\left(\frac{2k\pi + \theta}{n}\right) + i\sin\left(\frac{2k\pi + \theta}{n}\right)\right), k = 0,1,2, \dots, n - 1.$$
- Euler's formula: $e^{ix} = cosx + i\ sinx$
- Logarithm of a Complex number:

$$Log(x + iy) = \log\left(\sqrt{x^2 + y^2}\right) + i(2n\pi + \tan^{-1}\frac{y}{x})$$

Q-1a . $Express\ \sqrt{3} - i\ into\ polar\ form$

Q-1b. $Determine\ and\ sketch\ the\ region:\ (i) 0 \leq \arg z \leq \pi/4, (ii) |2z + 3| > 4,$
$Which\ of\ them\ are\ domains?$

Q-1c. $Find\ the\ moduli\ and\ argument\ of\ z = \frac{1-i}{1+i}.$

Q- 2. $State\ De'Movier's\ formula. Find\ arg\ \left[\frac{i}{-2-2i}\right]$

Q-3. $Find\ the\ real\ and\ imaginary\ parts\ of\ f(z) = \frac{3i}{2+3i}.$

Q-4.$State\ De - Movire's\ formula\ and\ hence\ evaluate\ \left(1 + i\sqrt{3}\right)^{100} + \left(1 - i\sqrt{3}\right)^{100}.$

Q-5. $If\ x_r = cos\frac{\pi}{2^r} + isin\frac{\pi}{2^r}, prove\ that\ x_1.x_2.x_3 \ldots\ldots \infty = -1.$

Q-6.$Find\ real\ and\ imaginary\ part\ of\ (-1-i)^7 + (-1+i)^7.$

Q-7.$Find\ the\ distinct\ values\ of\ z = (1+i)^{\frac{1}{3}}.$

Q-8. $Prove\ that\ the\ n^{th}\ roots\ of\ unity\ are\ in\ geometric\ progression. Also\ show\ that\ their\ sum\ is\ zero.$

Q-9.$If\ i^{i^{i\ldots\ldots\infty}} = A + iB, Prove\ that\ A^2 + B^2 = e^{-\pi(4n+1)B}.$

Q-10.$Separate\ \tan^{-1}(x+iy)\ into\ real\ and\ imaginary\ parts.$

Q-11.$Expand\ \sin^5\theta\cos^3\theta\ in\ terms\ of\ sines\ of\ multiples\ of\ \theta.$

Q-12. $Let\ z = x + iy.\ Showing\ details, find, in\ terms\ of\ x\ and\ y: (i)\ Im\left(\frac{1}{z}\right) (ii) Im\left(\frac{1}{\bar{z}^2}\right)$

Q-13. $Find\ all\ the\ values\ of\ \left(\frac{1}{2} + \frac{\sqrt{3}}{2}i\right)^{3/4}. Also\ find\ their\ product.$

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Grade: _____________________

Assignment 01

1. $Find\ and\ graph\ all\ roots\ in\ the\ complex\ plane.$
   $(i)\ \sqrt[5]{-1}\ \ (ii)\ \sqrt[4]{i}\ \ (iii)(1-i)^{1/3}\ (iii)\sqrt[3]{216}\ (iv)\ \sqrt[8]{1}$
2. $Find\ real\ and\ imaginar\ part\ of\ (i)e^z\ \ (ii)\sin z\ \ (iii)\sinh z\ \ (iv)\cosh 3 + 4i\ (v)e^{z^2}, e^{-\pi z}.$
3. $Let\ z = x + iy. Showing\ details, find, in\ terms\ of\ x\ and\ y: (i)\ Im\left(\frac{1}{z^2}\right)\ (ii)\ Re(1+i)^{16}z^2$
   $(iii) Re\left(\frac{z}{\bar{z}}\right),\ Im\left(\frac{z}{\bar{z}}\right)$
4. $Principal\ Value\ Ln\ z. Find\ Ln\ z\ when\ z\ equals\ (i)ei\ (ii)\ i\ (iii)3 - 4i$

Tutorial 02

Analytic Function & Cauchy-Riemann Equation

Formulas

- $Function\ of\ a\ Complex\ variable\ w = f(z) = u(x, y) + i\ v(x, y)$
- $C - R\ Equations:\ u_x = v_y\ \&\ u_y = -v_x$
- $C - R\ Equations\ in\ Polar\ form: \frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}\ \&\ \frac{1}{r}\frac{\partial u}{\partial \theta} = -\frac{\partial v}{\partial r}$
- $Laplace's\ Differential\ Equation:\ u_{xx} + u_{yy} = 0$

Q-1.$Check\ wether\ f(z) = \frac{z}{z-1}\ is\ contionous\ or\ not.$

Q-2. $State\ Cauchy - Riemann\ Equations. Show\ that$
$(I) f(z) = sin\ z\ is\ everywhere\ analytic\ (II)\ f(z) = xy + iy\ is\ nowhere\ analytic$

Q-3.$Show\ that\ f(z) = \bar{z}\ \&\ |z|\ are\ not\ analytic\ functions.$

Q-4.$Show\ that\ the\ function\ u = x^3 - 3xy^2\ is\ harmonic\ and\ find\ the\ corresponding\ analytic\ function.$

Q-5.$If\ u - v = \frac{e^y - \cos x + \sin x}{\cosh y - \cos x}, find\ an\ analytic\ function\ f(z) = u + iv\ with\ f\left(\frac{\pi}{2}\right) = \frac{3-i}{2}.$

Q-6.$Determine\ f(z) = u + iv\ , whose\ imaginary\ part\ is\ \log(x^2 + y^2) + x - 2y.$

Q-7. $Verify\ whether\ u(x, y) = x^2 - y^2\ is\ harmonic\ or\ not?\ Also\ find\ out\ its\ harmonic\ conjugate.$

Q-8. $Separate\ real\ and\ imaginary\ parts\ of\ f(z) = e^z\ . Also\ prove\ that\ it\ is\ analytic\ everywhere.$

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Grade: _____________________

Assignment II

1. $If\ u = 2x(1 - y), find\ an\ analytic\ function\ f(z) = u + iv$
2. $Separate\ real\ and\ imaginary\ parts\ of\ f(z) = ze^z\ . Also\ prove\ that\ it\ is\ analytic\ everywhere$
3. $Find\ the\ principal\ value. Show\ details. I. (2i)^{2i}\ II. (1 + i)^{1-i}\ III. (1 - i)^{1+i}$
4. $Write\ Cauchy - Riemann\ in\ polar\ and\ and\ cartiesian\ form$

Tutorial -03
Power Series & Radius of convergence

Formulas

- $Power\ series\ in\ powers\ of\ (x - x_0): \sum_{n=0}^{n=\infty} c_n (x - x_0)^n\ \ where\ x_0 = centre$
- $If\ the\ series\ \sum_{n=0}^{n=\infty} z_n\ \ converges, then\ \lim_{n\to\infty} z_n = 0$
- $Ratio\ Test: Let\ u_n\ be\ a\ positive\ term\ series\ and\ \lim_{n\to\infty} \frac{u_{n+1}}{u_n} = l,$

  $then\ (i) If\ l < 1\ then\ series\ converges$

  $(ii) If\ l > 1\ then\ series\ diverges$

  $(iii) If\ l = 1\ then\ test\ fails$
- $Interval\ of\ convergence: (x_0 - R, x_0 + R)$
- $Taylor\ series\ always\ converges\ for\ |z| < 1.$

Q-1. $Check\ the\ convergence\ of\ \sum_{n=1}^{n=\infty}(3 + 4i)^n z^n. Also\ find\ ROC\ \&\ IOC$

Q-2. $Check\ the\ convergence\ of\ \sum_{n=1}^{n=\infty}\left(\frac{(n!)^2}{(2n)!}\right) z^n.\ Also\ find\ ROC\ \&\ IOC$

Q-3. $Find\ the\ radius\ of\ convergence\ of\ \sum_{n=1}^{n=\infty}\left(\frac{6n+1}{2n+5}\right)(z - 2i)^n.$

Q-4. $Check\ the\ convergence\ of\ \sum_{n=1}^{n=\infty}\frac{n!}{n^2} z^n. Also\ find\ ROC\ \&\ IOC$

Q-5. $Check\ the\ convergence\ of\ \sum_{n=0}^{n=\infty}\frac{z^n}{n!}.\ Also\ find\ ROC\ \&\ IOC$

Remarks: ____

Signature of Faculty: __________________ **Date:** ______________________

Grade: __________________

Assignment III

1. $Find\ series\ expression\ for\ \tan^{-1} z\ \ at\ z = 0$
2. $Check\ the\ convergence\ of\ \sum_{n=1}^{n=\infty}\frac{(-1)^n}{n}(z - 2i)^n.\ Also\ find\ ROC\ \&\ IOC$

Tutorial 04
Conformal mapping & Bilinear(Mobius) Transform

Formulas

- $Conformal\ mappimng: f'(z) \neq 0\ at\ z = z_0.$
- $Identity\ mapping: w = z$
- $Translation: w = z + \propto$
- $Rotation: w = ze^{i\theta}$
- $Magnification: w = cz\ (c > 0)$
- $Inversion\ w = \frac{1}{z}$
- $Linear\ transformation\ w = az + b$
- $The\ Joukowski\ Transformation\ w = 1 + \frac{1}{z}$
- $w = e^z$
- $Cross\ ratio: \frac{(z_1-z_2)(z_3-z_4)}{(z_4-z_1)(z_2-z_3)}$
- $Bilinear\ transformation: w = \frac{az+b}{cz+d}\ where\ ad - bc \neq 0$
- $\frac{w-w_1}{w-w_3} \cdot \frac{w_2-w_3}{w_2-w_1} = \frac{z-z_1}{z-z_3} \cdot \frac{z_2-z_3}{z_2-z_1}$

Q-1. $Find\ the\ image\ of\ the\ infinite\ strip\ 0 < y < \frac{1}{2}\ under\ the\ transformation\ w = \frac{1}{z}.$

Q-2. $Show\ that\ the\ mapping\ by\ w = 1/z\ transforms\ circles\ and\ lines\ into\ circles\ and\ lines.$

Q-3. $Find\ the\ image\ of\ upper\ half\ of\ z - plane\ under\ transformation\ w = \frac{z}{1-z}.$

Q-4. $Find\ the\ image\ of\ semi - infinite\ strip\ x > 0, 0 < y < 2\ when\ w = iz + 1.$
$Sketch\ the\ strip\ and\ its\ image.$

Q-5. $Find\ the\ bilinear\ transformation\ that\ maps\ the\ points\ z_1 = -1, z_2 = 0, z_3 = 1$
$onto\ the\ points\ w_1 = -i, w_2 = 1, w_3 = i\ respectively. Also\ find\ w\ for\ z = \infty.$

Q-6. $Find\ the\ bilinear\ transformation\ which\ transforms\ z_1 = \infty, z_2 = i, z_3 = 0\ into\ w_1 = 0,$
$w_2 = i, w_3 = \infty.$

Q-7. $Determine\ the\ region\ of\ the\ w - plane\ into\ which\ the\ region\ bounded\ by\ x = 1,$
$y = 1, x + y = 1\ is\ mapped\ by\ w = z^2. Show\ that\ angles\ are\ preserved.$

Q-8. $Find\ and\ sketch\ the\ image\ of\ |z| = 2\ and\ |z| = 1\ of\ z - plane\ under\ the$
$transformation\ w = z + \frac{1}{z}.$

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Grade: ______________________

Assignment IV

1. Find the bilinear transformation which maps the points z = 1, i, – 1, in to the points $w = i,\ 0, 1$
2. $Determine\ the\ mobius\ transformation\ that\ maps\ z_1 = 0, z_2 = 1,\ z_3 = \infty\ onto\ w_1 = -1,$ $w_2 = -i, w_3 = 1\ respectively$
3. $Find\ the\ fixed\ points\ of\ \ w = \frac{iz+4}{2z-5i}.$

Tutorial -05
Complex integration

Formulas $Line\ integral\ in\ complex\ plane: \int_c f(z)dz$

- $Parametrization\ method: \int_c f(z)dz = \int_a^b f[z(t)]z'(t)dt$
- $Cauchy - Integral\ Theorem: \oint_c f(z)dz = 0\ where\ a\ point\ z_0\ is\ lying\ outside\ C$
- $Cauchy\ Integral\ Formula: \oint_c \frac{f(z)}{z-z_0} dz = 2\pi i f(z_0)\ \ where\ a\ point\ z_0\ \ is\ lying\ inside\ C$

Q-1. $Evaluate\ \int_{z=0}^{z=1+i} \{(3x^2+4xy+3y^2)dx + 2(x^2+3xy+4y^2)dy\}\ \ (i) Along\ x = y^2 (ii)\ Along\ y = x^2$

Q-2. $Evaluate\ \int_c Re(z^2)dz\ \ where\ C\ is\ the\ boundary\ of\ the\ square\ with\ vertices\ 0, i, 1+i, 1\ in\ the\ clockwise\ direction.$

Q-3. $Evaluate \int z^2\ dz\ along\ the\ line\ joining\ the\ points\ (0,0) and\ (2,1).$

Q-4. $Evaluate \int |z|^2\ dz\ \ where\ C\ is\ the\ boundary\ of\ the\ square\ with\ vertices\ (0,0), (1,0), (1,1), (0,1)$

Q-5. $Find\ \ I = \oint \frac{cosz}{\left(z-\frac{\pi}{4}\right)^3}\ dz\ \ , where\ C: \left|Z - \frac{\pi}{2}\right| = 1$

Q-6. $Evaluate\ \oint_c [\frac{3}{z-i} - \frac{6}{(z-i)^2}]\ dz, where\ C: |z| = 2.$

Q-7. $Evaluate\ \oint_c \frac{e^z}{z^2+1} d\,z\ over\ the\ circle\ |z| = 2$

Q-8. $Evaluate\ \oint_c \frac{z^3-z^2+z-1}{z^3+4z} dz\ \ counter\ clockwise\ around\ C, where\ C\ is\ |z| = 1\ and\ |z| = 3.$

Q-9. $Evaluate \oint_c \frac{z}{z^2+1} dz\ ,\ \ where\ C\ is\ the\ circle\ (i)|z+1/z| = 2\ (ii)|z+i| = 1$

Q-10. $Evaluate\ \oint_c \frac{z}{(9-z^2)(z+i)} dz\ , where\ C\ is\ the\ circle\ |z| = 2$

Q-11. $Evaluate\ \oint_c \frac{1}{(z-1)^2(z-3)} dz, where\ C: |z| = 2$

Q-11. $Evaluate\ \oint_c \frac{e^z}{z(1-z)^3} dz\ \ where\ C\ is\ (i)|z| = \frac{1}{2}\ \&\ (ii)|z-1| = \frac{1}{2}$

Q-12. $State\ the\ Cauchy's\ Integral\ Formula\ and\ its\ extension. Hence\ evaluate\ integral$
$\oint_c \frac{(1+z)}{z^2-2z+5} dz, where\ C\ is\ circle\ |z\ +1\ +\ i\ |\ = 2.$

Remarks: ____
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Grade: ______________________

Assignment V

1. $Evaluate\ \oint_c \frac{e^{2z}}{z - \ln 1.5} dz\ , where\ C\ is\ the\ square\ with\ vertices\ (1,0), (0,1), (0-1)\ and\ (-1,0)$
2. $Verify\ whether\ u(x,y) = e^x \cos x + x\ is\ harmonic\ or\ not?\ Also\ find\ out\ its\ harmonic\ conjugate.$
3. $Evaluate\ \oint_c \frac{z^2+1}{z^2-1}\ dz\ , if\ C\ is\ the\ circle\ of\ unit\ radius\ with\ center\ (I) at\ z = 1\ and\ (II) at\ z = -1.$

Tutorial 06

Taylor series & Laurent Series

Formulas

- $\frac{1}{1-z} = \sum_{n=0}^{n=\infty} z^n$
- $\frac{1}{1+z} = \sum_{n=0}^{n=\infty} (-1)^n z^n$
- $Laurent\ series: f(z) = \sum_{n=0}^{n=\infty} a_n (z - z_0)^n\ ,\ \ r_1 < |z - z_0| < r_2$
  $where\ a_n = \frac{1}{2\pi i} \oint \frac{f(z)dz}{(z - z_0)^n}, n = 0, \pm 1, \pm 2, \ldots..$

Q-1. Expand $\mathrm{f(z)} = \frac{\mathrm{z}-1}{\mathrm{z}+1}$ as a Taylor's series about the point $\mathrm{z} = 0$

Q-2. $Expand\ \frac{1}{z(z^2-3z+2)}\ about\ z = 0\ , for\ the\ region$
$(i) 0 < |z| < 1\ (ii) 1 < |z| < 2\ (iii) |z| > 2$

Q-3. $Give\ two\ Laurent\ series\ expansions\ in\ powers\ of\ z\ for\ the\ function\ f(z) = \frac{1}{[z^2\ (1 - z)] and}$
$specify\ the\ regions\ in\ which\ those\ expansions\ are\ valid.$

Q-4. $Expand\ f(z) = \frac{1}{(z+2)(z+4)}\ \ in\ the\ region\ (i) |z| < 2\ (ii) 2 < |z| < 4\ (iii) |z| > 4$

Q-5. $Expand\ f(z) = \frac{z+3}{z(z^2-z-2)}\ in\ the\ region\ (i) |z| > 1\ (ii) 1 < |z| < 2\ (iii) |z| > 2$

Q-6. $Expand\ f(z) = \frac{1}{z(1-z)}\ \ in\ the\ region\ (i) |z + 1| < 1\ (ii) 1 < |z + 1| < 2\ (iii)\ |z + 1| > 2$

Remarks: ____

Signature of Faculty: ___________________ **Date:** _______________________

Grade: _____________________

Assignment VI

Q-1. $Expand\ f(z) = \frac{1}{z(z-1)}\ in\ the\ region\ (i) 0 < |z| < 1\ (ii) 0 < |z - 1| < 1$

Q-2. $Find\ Laurent's series\ expansion\ of\ f(z) = \frac{1}{6-z-z^2}\ in\ (i) domian\ |z| < 2 (ii) domain$
$2 < |z| < 3\ (iii) |z| > 3\ .$

Tutorial 07

Singularity & Residue

Formulas

- $f(z) = \sum_{n=0}^{n=\infty} a_n(z-z_0)^n = a_0 + a_1(z-z_0) + \cdots + \frac{a_{-1}}{(z-z_0)} + \frac{a_{-2}}{(z-z_0)^2} + \cdots$
  that converges for all point near $\mathrm{z} = \mathrm{z}_0$ which is valid in some
  $deleted\ neighbourhood\ 0 < |Z - Z_0| < R$
  $\boldsymbol{a_{-1} = RES_{Z=Z_0} f(z)\ \ OR\ Res(f(z), z_0)}$
- $Residue\ at\ a\ simple\ pole: Res(f(z), z_0) = \lim_{z\to z_0}(z-z_0)f(z)$
- $Residue\ at\ pole\ of\ order\ n: Res(f(z), z_0) = \frac{1}{(n-1)!}\lim_{z\to z_0}(z-z_0)^n f(z)$

Q-1. $Classify\ the\ singularities\ of\ the\ analytic\ function. In\ each\ of\ the\ following\ case, identify\ the\ singular\ point\ and\ its\ type\ with\ justification.\ (i)\ \dfrac{z^2}{z+1}(z\ +\ 1)\ (ii)\ \dfrac{\sin z}{z}\ \ (iii)\ \dfrac{1\ -\ cosh\ z}{z^3}$

Q-2. $Find\ the\ residues\ of\ f(z) = \frac{z^2-2z}{(z+1)^2(z^2+4)}\ at\ each\ of\ its\ poles\ in\ the\ finite\ z-plane.$

Q-3. $Find\ residue\ of\ f(z) = \frac{1}{(z-1)^2(z-3)}; Res(f(z), 1)$

Q-4. $Find\ residue\ for\ f(z) = \frac{5z^2-4z+3}{(z+1)(z+2)(z+3)}\ at\ each\ of\ its\ poles.$

Q-5. $Evaluate\ \int_c \frac{z^2 dz}{(z-1)^2(z+2)}\ where\ C: |Z| = 3\ by\ residue\ theory$

Q-6. $Evaluate \int_c \frac{e^{2z}dz}{(z+1)^3}\ where\ C\ is\ the\ ellipse\ 4x^2 + 9y^2 = 16.$

Q-7. $Evaluate\ \int_0^{2\pi} \frac{cos3\theta}{5-4cos\theta} d\theta$

Q-8. $Using\ the\ residue\ theorem, evaluate\ \int_0^{2\pi} \frac{d\theta}{5-3\sin\theta}.$

Q-9. $Evaluate\ \int_0^{2\pi} \frac{d\theta}{5-3sin\theta}.$

Q-10. $Evaluate\ \int_{-\infty}^{\infty} \frac{dx}{(x^2+1)(x^2+4)}.$

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Grade: ______________________

Tutorial 08

Partial Differential Equations

1. *Find particular integral of* $4r + 12s + 9t = e^{3x-2x}$.

2. *Find partial differential equation of i.* $z = ax^2 + by^2$

ii. $z = (x+a)(y+b)$ *iii.* $z = z = (x-a)^2 + (x-b)^2$

iv. Form the partial differential equation by eliminating the arbitrary function from $f(x^2 + y^2, z - xy) = 0$.

3. *Find Complete integral of* $(p+q)(z - px - qy) = 1$.

4. *Determine complete integral of* $p^2 y(1 + x^2) = qx^2$.

5. *Determine complete integral of* $pq = 1$.

6. *Determine complete integral of* $(D^3 - 3D^2D' + 2DD'^2)z = 0$.

Remarks: ____

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Grade: ____________________

Assignment VIII

1. *Find partial differential equation of i.* $z = (x^2 + a)(y^2 + b)$

   ii. $z = ax + by + ab$

2. *Determine complete integral of* $px + qy = pq$.

Tutorial 09

Partial Differential Equations, Heat & Wave Equations

Q-1. $Solve\ 3u_x + 2u_y = 0, where\ u(x\ ,0) = \ 4e^{-x}\ using\ the\ method\ of\ separation\ of\ variables.$

Q-2. A bar with insulated sides is initially at temperature 0^0C , throughout. The end $x = \ 0$ $is\ kept\ at\ 0^0C\ and\ heat\ is\ suddenly\ applied\ at\ the\ end\ x = \ l\ \ so\ that\ u_x = A\ for\ x = \ l,$ $where\ A\ is\ a\ constant. Find\ the\ \ temperature\ function.$

Q-3. $Solve\ the\ partial\ differential\ equation\ using\ method\ of\ separation\ of\ variables.$
$u_x \ = \ 4u_y\ ;\ u(0,y) \ = \ 8e \ -3y\ \ .$

Q-3. $A\ tightly\ stretched\ string\ of\ length\ l\ with\ fixed\ ends\ is\ initially\ in\ equilibrium\ position.$ $It\ is\ set\ vibrating\ by\ giving\ each\ point\ a\ velocity\ v_0 \sin^3\frac{\pi x}{l}. Find\ the\ displacement\ y(x,t).$

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Grade: ____________________

Assignment IX

Q-1. $Find\ the\ solution\ of\ the\ wave\ equation$ $u_{tt} = c^2u_{xx}$, $0 \leq x \leq L$ satisfying the conditions:
$u(0,t) = u(L,t) = 0, u_t(x,0) = 0, u(x,0) = \frac{x\pi}{L}, 0 < x < L.$

Q-2. $Using\ the\ method\ of\ separation\ of\ variables\ solve, u_x = 2u_t + u\ where\ u(x,0) = 6e^{-3x}$.

Reference Books:

1. *Erwin Kreyszig, Advanced Engineering Mathematics, 8th Edition, John Wiley and Sons*
2. *Ian Sneddon, Elements of Partial Differential Equations, McGraw Hill.*
3. *G. F. Simons, Differential Equations, Tata MacGraw Hill, New Delhi, 1972.*
4. *J. W. Brown and R. V. Churchill, Complex Variables and Applications, McGraw Hill.*
5. *S.Ponnusamy, Foundation of Complex analysis, Narosa publication, 1995.*
6. *T. Amaranath, Partial Differential Equations, Narosa Publ.*

Useful link:

- https://ocw.mit.edu/resources/res-18-008-calculus-revisited-complex-variables-differential-equations-and-linear-algebra-fall-2011/part-i/
- https://ocw.mit.edu/resources/res-18-008-calculus-revisited-complex-variables-differential-equations-and-linear-algebra-fall-2011/part-ii/
- https://ocw.mit.edu/courses/mathematics/18-03sc-differential-equations-fall-2011/